Calculus III (Math 233) Exam 1

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Justify answers and show all work for full credit.

NAME: ________________________________

Problem 1. Let \( \vec{u} = \langle 4, 4, 5 \rangle \) and \( \vec{v} = \langle 2, -1, 1 \rangle \).

(a) Find a unit vector in the direction of \( \vec{v} \).

(b) Find \( ||\text{proj}_\vec{v} \vec{u}|| \).

(c) Express \( \vec{u} \) as the sum of \( \vec{m} = \vec{u}|| \) parallel to \( \vec{v} \), and \( \vec{n} = \vec{u}_\perp \) orthogonal to \( \vec{v} \).

Problem 2. Consider three points \( A(-2, 1, -1), B(1, 2, 2), C(1, 1, 5) \).

(a) Are the points \( A, B, C \) collinear? Justify your answer using the cross-product.

(b) Find the area of the triangle formed by \( A, B, C \).

(c) Find the equation of the plane that contains \( A, B, C \).

Problem 3. Consider two points \( E(1, 0, 1), F(-3, 2, 3) \).

(a) Find a parametric equation of the line through \( E \) and \( F \).

(b) Find the symmetric equation of the line through \( E \) and \( F \).

(c) Find the cylindrical coordinates for \( E \).

(d) Find the spherical coordinates for \( E \).

Problem 4.

(a) Find the angle between the planes \( x - y = 3 \) and \( -y + z = 1 \).
   (Hint: Angle between the planes is the angle between their normal vectors.)

(b) Find the equation of the plane that passes through the point \( (1, 2, -1) \) and is perpendicular to the line \( x - 2 = \frac{y + 1}{2} = \frac{z}{4} \).
Problem 5. For each equation below, find the surface in \( \mathbb{R}^3 \) that matches it.

(a) \( x^2 + 4y^2 + 4z^2 = 16 \)

(b) \( 4x^2 + y^2 + 4z^2 = 16 \)

(c) \( z = 9x^2 + 4y^2 \)

(d) \( z = 9x^2 - 4y^2 \)

(e) \( 9x^2 + 4y^2 = 2z^2 + 72 \)

(f) \( 9x^2 + 4z^2 = 2y^2 - 72 \)

(g) \( 9x^2 + 4y^2 = 2z^2 \)

(h) \( 9x^2 - 4y^2 = 72 \)