Linear Algebra (Math 338) Sample Midterm Exam 2

Date: November 22, 2005 Professor Ilya Kofman

1. Justify answers and show all work for full credit, except for Problem 1.
2. No *symbolic* calculators allowed on this exam.
3. Answer the questions in the space provided on the question sheet. If you run out of room for an answer, continue on back of the page.

NAME: ____

1. ____________

2. ____________

3. ____________

4. ____________

5. ____________

6. ____________

7. ____________

8. ____________

Σ ____________

Problem 1. *(10 pts.) SHORT ANSWERS – NO PARTIAL CREDIT*

(a) Circle one: *Every spanning set of $\boldsymbol{R}^n$ has AT LEAST / AT MOST n vectors.*

(b) Circle one: *For $A_{m\times n}$, if rank$(A) < n$, then $Ax = 0$ has 0 / 1 / ∞-many nontrivial solutions.*

(c) *If the columns of $A_{n\times n}$ are an orthogonal set, then what are the possible values for rank(A)?*

(d) *If V has basis S, and T is obtained from S by the Gram-Schmidt process, what are the properties of T that are possibly different from S?*

(e) *Let A be a 4×3 matrix such that the sum of its columns equals 0. What is the largest possible value for the row rank of A?*

Problem 2.

$$S = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 4 \\ 1 \end{bmatrix} \right\}.$$

Find a subset of S that is a basis for $V = span(S)$.

Problem 3. *Let* $S = \{u_1, u_2, u_3\}$ *be a basis for* $\boldsymbol{R}^3$*, where*

$$u_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, u_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, u_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

(a) *Find the coordinate vector of* $v = \begin{bmatrix} 1 \\ 5 \\ 3 \end{bmatrix}$ *with respect to the basis* S.

(b) *If we start the Gram-Schmidt Process with* $v_1 = u_1$*, what is the second vector* v_2*?*

Problem 4.

$$A = \begin{bmatrix} 1 & 2 & 4 & 1 & 6 \\ 2 & 1 & 1 & 1 & 3 \\ -1 & 4 & 10 & 1 & 12 \end{bmatrix}$$

(a) *Find the rank and nullity of A. Justify!*

(b) *Find a basis for the orthogonal complement of the null space of A.*

Problem 5. *Let P_4 be the vector space of polynomials of degree ≤ 4.*

(a) *Write down a basis for P_4.*

(b) *Is the set $\{t^4 + 1,\ t^3 + t,\ t^2\}$ linearly independent? Justify.*

(c) *What is the orthogonal complement to P_3 in P_4?*

Problem 6. *Let S and T be bases for $\boldsymbol{R}^2$, where* $S = \left\{ \begin{bmatrix} 1 \\ 5 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \end{bmatrix} \right\}$.

(a) *Find the coordinate vector of* $v = \begin{bmatrix} -1 \\ 8 \end{bmatrix}$ *with respect to S.*

(b) *Find T, given that* $P_{S \leftarrow T} = \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$.

Problem 7. *Let* $L : \boldsymbol{R}^2 \to \boldsymbol{R}^2$ *be defined by* $L(x, y) = (x - 2y,\ x + 2y)$.
Let $S = \left\{ \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$ *be a basis for* $\boldsymbol{R}^2$*, and let* T *be the natural basis for* $\boldsymbol{R}^2$.

(a) *Find the matrix for* L *with respect to* T.

(b) *Find the matrix for* L *with respect to* S.

(c) *Find the rank and nullity of* L.