Linear Algebra (Math 338) Sample Midterm Exam 1

Date: October 6, 2005 | Professor Ilya Kofman

1. Justify answers and show all work for full credit, except for Problem 1.
2. No calculators allowed on this exam.
3. Answer the questions in the space provided on the question sheet. If you run out of room for an answer, continue on back of the page.

NAME: ____

1. ___________

2. ___________

3. ___________

4. ___________

5. ___________

6. ___________

7. ___________

Σ ___________

Problem 1. *(10 pts.) State whether the following statements are* **Always** *true,* **Sometimes** *true, or* **Never** *true. Please circle one of* **A, S, N** *below.*

(a) *An invertible matrix can be written as a product of symmetric matrices.*

A ***S*** ***N***

(b) *A homogeneous* 3×5 *linear system has a nontrivial solution.*

A ***S*** ***N***

(c) *If* $\det(A) = 0$*, then* $\det(A + B) = \det(B)$.

A ***S*** ***N***

(d) *If* $\det(A) = 0$*, then* $\det(BA) = 0$.

A ***S*** ***N***

(e) *A square matrix which has two identical columns is invertible.*

A ***S*** ***N***

Problem 2. *(15 pts.) Justify three out of the following four statements with a short general argument:*

(a) *If A is a non-singular $n \times n$ matrix then:*

$$\det\left(A^{-1}\right) \;=\; \frac{1}{\det(A)}.$$

(b) *If A and B are non-singular $n \times n$ matrices, then AB is also non-singular.*

(c) *A non-singular matrix has a unique inverse.*

(d) *If A and B are symmetric matrices, then AB is also symmetric.*

Problem 3. *(20 pts.) Write "impossible" or give an example of:*

(a) *A* 3×3 *matrix with no zeros but which is not invertible.*

(b) *A system with two equations and three unknowns that is inconsistent.*

(c) *A system with two equations and three unknowns that has a unique solution.*

(d) *A system with two equations and three unknowns that has infinitely many solutions.*

Problem 4. *(10 pts.) Consider the following linear system:*

$$\begin{cases} 2x_1 - x_2 + x_4 = 0 \\ -x_1 + 2x_2 - x_3 = 1 \\ -x_2 + 2x_3 = 0 \end{cases} .$$

Write its associated augmented matrix. Reduce the matrix to its row-echelon form. Use the procedure to solve the system.

Problem 5. *(25 pts.)*

(a) *Let:*

$$A = \begin{pmatrix} 2 & 1 & 3 \\ 0 & -1 & 5 \\ -4 & 2 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 2 & 0 & 0 \\ -1 & 1 & 3 \\ 5 & 1 & 0 \end{pmatrix}.$$

Compute $A + B$, AB, B^t, $\det(A)$, $\det(A^t)$ *and* $\det(3A)$.

(b) *Use elementary operations to find the inverse of:*

$$C = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{pmatrix}.$$

Problem 6. *(10 pts.) Use Cramer's rule to solve the following linear system:*

$$\begin{cases} 2x + y = 1 \\ x + 2y + z = 0 \\ y + 2z = 0 \end{cases} .$$

Problem 7. *(20 pts.)*

(a) *Let* $L : \boldsymbol{R}^3 \to \boldsymbol{R}^3$ *be the linear transformation defined by* $L(\boldsymbol{x}) = A\boldsymbol{x}$*, where*

$$A = \begin{pmatrix} 1 & 2 & 0 \\ 2 & -1 & 5 \\ 3 & 2 & 4 \end{pmatrix}.$$

Is the vector $(1, 2, 3)$ *in the range of* L*?*

(b) *Let* $L : \boldsymbol{R}^2 \to \boldsymbol{R}^3$ *be defined by* $L(x, y) = (2x + 3y, -2x + 3y, x + y)$*. Find the standard matrix representing* L*.*