# Linear Algebra (Math 338) Midterm Exam 3

Date: May 15, 2007 Professor Ilya Kofman

**NAME:** ________________________________________

**Problem 1.** *Let $S = \{u_1, u_2, u_3\}$ be a basis for $\mathbf{R}^3$, where*

$$u_1 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, \quad u_2 = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}, \quad u_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

**(a)** *Start the Gram-Schmidt Process with $v_1 = u_1$. Find $v_2$ and $v_3$.*

**(b)** *What are the vectors $\{w_1, w_2, w_3\}$ of the orthonormal basis?*

**Problem 2.**

**(a)** *Let $V = \operatorname{span}\{(-1,1,2),\ (2,4,-2)\}$ be a subset of $\mathbf{R}^3$. Find a basis for $V^\perp$. Hint: Use a matrix.*

**(b)** *If $A^2 = A^T$, what are the possible real eigenvalues of $A$? Justify.*

**Problem 3.** $A = \begin{bmatrix} 3 & 1 \\ -5 & -3 \end{bmatrix}$

**(a)** *Find the eigenvalues of $A$.*

**(b)** *Diagonalize $A$ (i.e, find $P$ and $D$ such that $P^{-1} A P = D$).*

**(c)** *Use part (b) to find the exact formula for $A^k$.*