# Linear Algebra (Math 338) Midterm Exam 2

Date: April 19, 2007 Professor Ilya Kofman

Justify answers and show all work for full credit, except for Problem 1.

**NAME:** ______________________________________________

1. ____________

2. ____________

3. ____________

4. ____________

5. ____________

6. ____________

$\Sigma$ ____________

**Problem 1.** *(10 pts.) SHORT ANSWERS – NO PARTIAL CREDIT*

**(a) Circle one:** *A linearly independent set in* $\boldsymbol{R}^n$ *that is NOT a basis has how many vectors?*

*AT LEAST* $n$ / *AT LEAST* $n+1$ / *AT MOST* $n-1$ / *AT MOST* $n$

**(b) Circle one:** *A spanning set in* $\boldsymbol{R}^n$ *that is NOT a basis has how many vectors?*

*AT LEAST* $n$ / *AT LEAST* $n+1$ / *AT MOST* $n-1$ / *AT MOST* $n$

**(c) Circle one:** *For* $A_{7\times 4}$, *if the columns of* $A$ *are linearly independent, then* $Ax = 0$ *has* $0$ / $1$ / $\infty$*-many solutions.*

**(d)** *If the rows of* $A_{3\times 5}$ *are linearly independent, what is* $\text{nullity}(A)$*?*

**(e)** *If* $\text{Im}(A_{3\times 5})$ *is the plane* $x - 4y + z = 0$, *what are* $\text{rank}(A)$ *and* $\text{nullity}(A)$*?*

**Problem 2.** *(15 pts.)*

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 7 & 1 \\ 2 & 5 & -1 \end{bmatrix}$$

**(a)** *Find the reduced row-echelon form, rref(A).*

**(b)** *Find the rank and nullity of A. Justify!*

**(c)** *Find a basis for the column space of A.*

**Problem 3.** *(15 pts.)* $A = \begin{bmatrix} 1 & 3 & -2 & 1 & 1 \\ 2 & 6 & -1 & -1 & 2 \\ 2 & 6 & 1 & -3 & -1 \end{bmatrix}$ $\quad \text{rref}(A) = \begin{bmatrix} 1 & 3 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

**(a)** *Find the rank and nullity of $A$.*

**(b)** *Find a basis for $Im(A)$.*

**(c)** *Find a basis for $Ker(A)$.*

**Problem 4.** *(15 pts.)*

**(a)** *Let* $L : \boldsymbol{R}^2 \to \boldsymbol{R}^3$ *be the linear transformation defined by* $L(\boldsymbol{x}) = A\boldsymbol{x}$, *where*

$$A = \begin{pmatrix} 2 & 1 \\ 3 & -2 \\ -1 & 3 \end{pmatrix}.$$

*Is the vector* $(-1, -2, 2)$ *in the range of* $L$*? Justify.*

**(b)** *Let* $L(x, y, z) = (5x + 3y - 2z,\ 4x - y + 3z)$.

*Find the standard matrix that represents* $L$.

**Problem 5.** *(25 pts.) Let* $S = \{u_1, u_2\}$ *and* $T = \{u_1, u_3\}$ *be two bases for* $\boldsymbol{R}^2$*, where*

$$u_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad u_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad u_3 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

**(a)** *Verify that* $S$ *is a basis.*

**(b)** *Find the coordinate vector of* $v = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$ *with respect to the basis* $S$.

**(c)** *Find the coordinate vector of* $v = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$ *with respect to the basis* $T$.

**(d)** *Find the transition matrix* $P_{T \leftarrow S}$.

**(e)** *Verify that* $[v]_S$ *and* $[v]_T$ *are related by the transition matrix.*

**Problem 6.** *(20 pts.)*

**(a)** *A student's scores are given as a vector* $\mathbf{u}$*, and each score is worth a certain percentage of the total, given by a vector* $\mathbf{v}$*. For example,* $\mathbf{u} = (82, 75, 89, 9, 6, 8, 8, 7, 9)$ *and* $\mathbf{v} = (20, 20, 30, 5, 5, 5, 5, 5, 5)$*. Write a general formula for the total student score* $S(\mathbf{u}, \mathbf{v})$ *out of* $100$*. (Use* $\mathbf{u}$ *and* $\mathbf{v}$ *in general, not just for this example!)*

**(b)** *Suppose* $\mathbf{u}$ *is orthogonal to both* $\mathbf{v}$ *and* $\mathbf{w}$*. Show that* $\mathbf{u}$ *is orthogonal to any vector* $(r\mathbf{v} + s\mathbf{w})$ *for any scalars* $r$ *and* $s$*.*

**(c)** *Write the matrix for a linear transformation of* $\boldsymbol{R}^2$ *that makes a* $90^\circ$ *rotation and doubles the length. (Hint: Draw a picture.)*

**(d)** *Write the matrix for a linear transformation of* $\boldsymbol{R}^3$ *that makes a* $90^\circ$ *rotation in the* $xz$*–plane.*