Linear Algebra (Math 338) Midterm Exam 1

Date: March 1, 2007 Professor Ilya Kofman

1. Justify answers and show all work for full credit, except for Problem 1.
2. No *symbolic* calculators allowed on this exam.
3. Answer the questions in the space provided on the question sheet. If you run out of room for an answer, continue on back of the page.

NAME: ____

1. ___________

2. ___________

3. ___________

4. ___________

5. ___________

6. ___________

7. ___________

Σ ___________

Problem 1. *(10 pts.) State whether the following statements are* **Always** *true,* **Sometimes** *true, or* **Never** *true. Please circle one of* **A, S, N** *below.*

(a) *If A and B are diagonal matrices then $AB = BA$.*

A ***S*** ***N***

(b) *If A and B are $n \times n$ matrics and A is singular, then (AB) is singular.*

A ***S*** ***N***

(c) *If $A^2 = I_n$, then $A = I_n$ or $A = -I_n$.*

A ***S*** ***N***

(d) *Let O be the zero matrix. If $\mathrm{rref}(A) = O$, then $A = O$.*

A ***S*** ***N***

(e) *A homogeneous system with more variables than equations has a finite number of solutions.*

A ***S*** ***N***

Problem 2. *(15 pts.) Justify three out of the following four statements with a short general argument. (Do all four for a bonus.)*

(a) *If* $A^{-1} = A^T$ *then* $\det\left(A^{-1}\right) = \pm 1$.

(b) *If* A *is any* $n \times n$ *matrix then* $(A + A^T)$ *is symmetric.*

(c) *If* $\det(A) \neq 0$ *then* $A\boldsymbol{x} = \mathbf{b}$ *has a unique solution.*

(d) *If* $A\boldsymbol{x} = \mathbf{b}$ *has solutions* $\boldsymbol{u}_1 = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}$ *and* $\boldsymbol{u}_2 = \begin{pmatrix} 4 \\ 4 \\ 4 \end{pmatrix}$ *then* $\boldsymbol{u}_3 = \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix}$ *is also a solution.*

Problem 3. *(15 pts.)*

(a) *Give an example of a* 3×4 *matrix in reduced row-echelon form* (rref) *that has one row* $[0\ 0\ 0\ 1]$ *and has two entries "2".*

(b) *Solve the following linear system:*

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ 3 \end{bmatrix}$$

(c) *If* A *is an invertible matrix such that* $A^2 = A$, *compute the determinant* $|A|$. *Show your work!*

Problem 4. *(15 pts.) Evaluate the following determinants:*

(a) $$\begin{vmatrix} 0 & 0 & 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & 0 & 5 & 6 \\ 0 & 0 & 0 & 4 & 5 & 6 \\ 0 & 0 & 3 & 4 & 5 & 6 \\ 0 & 2 & 3 & 4 & 5 & 6 \\ 1 & 2 & 3 & 4 & 5 & 6 \end{vmatrix}$$

(b) *If A, B are 3×3 matrices with $|A| = 2$ and $|B| = 3$, compute $|2AB|$.*

(c) *If A, B are 3×3 matrices with $|A| = 2$ and $|B| = 3$, compute $|A^4 B^T A^{-1}|$.*

Problem 5. *(15 pts.) Consider the following linear system:*

$$\begin{cases} x_1 - x_2 + x_4 = 2 \\ x_1 - x_3 + 2x_4 = 0 \\ -x_2 + x_3 + x_4 = -6 \end{cases} .$$

(a) *Write its associated augmented matrix.*

(b) *Reduce the matrix to its reduced row-echelon form* (rref)*.*

(c) *Use this procedure to solve the system.*

Problem 6. *(15 pts.)*

$$A = \begin{pmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & a & 2 \end{pmatrix}$$

(a) *For which values of a is A invertible?*

(b) *Use elementary operations to find the inverse of A when $a = -1$.*

Problem 7. *(15 pts.) Use Cramer's rule to solve the following linear system:*

$$\begin{cases} x - z = -3 \\ 2x + y = 2 \\ -2y - z = -1 \end{cases} .$$