Nonlinear Days in New York

List of speakers:

Thomas Bartsch (Giessen)
Vieri Benci (Pisa)
Lucio Boccardo (Roma)
Alice Chang (Princeton)
Sagun Chanillo (Rutgers)
Marco Degiovanni (Brescia)
Fanghua Lin (Courant)
Andrea Malchiodi (Pisa)
Jie Qing (Santa Cruz)
Ovidiu Savin (Columbia)
Andrzej Szulkin (Stockholm)
Susanna Terracini (Torino)

Workshop organized by:

Z. Huang, M. Lucia
(The City University of New York, USA)
M. Squassina
(Università Cattolica del Sacro Cuore, Italy)

CITY UNIVERSITY OF NEW YORK
THE GRADUATE CENTER

ITS @ The Graduate Center
Initiative for the Theoretical Sciences

Nonlinear days in New York

April 25th-27th, 2018
Science Center, Room 4102
Graduate Center, CUNY

Wednesday April 25th

- *9 am - 9:30 am*: Breakfast
- *9:30 am - 10:30 am*: Lucio Boccardo
- *10:30 am - 10:45 am*: Coffee
- *10:45 am - 11:45 am*: Fanghua Lin

- *1:00 pm - 2:00 pm*: Ovidiu Savin
- *2:00 pm - 2:15 pm*: More Coffee
- *2:15 pm - 3:15 pm*: Marco Degiovanni

Thursday April 26th

- *9 am - 9:30 am*: Breakfast
- *9:30 am - 10:30 am*: Alice Chang
- *10:30 am - 10:45 am*: Coffee
- *10:45 am - 11:45 am*: Vieri Benci

- *1:00 pm - 2:00 pm*: Jie Qing
- *2:00 pm - 2:15 pm*: More Coffee
- *2:15 pm - 3:15 pm*: Thomas Bartsch

Friday April 27th

- *9 am - 9:30 am*: Breakfast
- *9:30 am - 10:30 am*: Andrea Malchiodi
- *10:30 am - 10:45 am*: Coffee
- *10:45 am - 11:45 am*: Susanna Terracini

- *1:00 pm - 2:00 pm*: Andrzej Szulkin
- *2:00 pm - 2:15 pm*: More Coffee
- *2:15 pm - 3:15 pm*: Sagun Chanillo

Abstracts

Thomas Bartsch (Universität Giessen) **A natural constraint approach to normalized solutions of nonlinear Schrödinger equations and systems**

The talk will be concerned with the existence of solutions $\lambda_1, \lambda_2 \in \mathbb{R}$, $u, v \in H^1(\mathbb{R}^3)$ *to the system*

$$\begin{cases} -\Delta u + \lambda_1 u = \mu_1 u^3 + \beta u v^2 & \text{in } \mathbb{R}^3 \\ -\Delta v + \lambda_2 v = \mu_2 v^3 + \beta u^2 v & \text{in } \mathbb{R}^3 \end{cases}$$

with prescribed L^2 *norms*

$$\int_{\mathbb{R}^3} u^2 = a_1^2 \quad \text{and} \quad \int_{\mathbb{R}^3} v^2 = a_2^2.$$

Here $\mu_1, \mu_2, a_1, a_2 > 0$ *and* $\beta < 0$ *are prescribed. We present a new approach that is based on the introduction of a natural constraint associated to the problem. Our method can be adapted to other problems with normalization constraints. In particular it leads to alternative and simplified proofs of some results on scalar nonlinear Schrödinger equations already available in the literature. This is joint work with Nicola Soave.*

Vieri Benci (Università di Pisa), **TBA**

Lucio Boccardo ("Sapienza" Università di Roma), **Regularizing effect of the lower order terms in some nonlinear Dirichlet problems problems**

In this talk Ω *is a bounded, open subset of* $\mathbb{R}^N$, *with* $N > 2$, $f \in L^m(\Omega)$ *with* $m \geq 1$, $M(x)$ *is a measurable matrix such that*

$$\alpha|\xi|^2 \leq \langle M(x)\,\xi,\,\xi\rangle\,, \qquad |M(x)| \leq \beta\,,$$

for almost every x *in* Ω, *and for every* ξ *in* $\mathbb{R}^N$, *with* $0 < \alpha \leq \beta$. *We discuss semilinear Dirichlet problems of the type*

$$-\operatorname{div}\big(M(x)\,\nabla u\big) + H(f(x), u) = 0,$$

where the solution u *is more regular than the solution* w *of the linear problem*

$$-\operatorname{div}\big(M(x)\,\nabla w\big) = f(x).$$

Results by R. Cirmi, L. Orsina, D. Arcoya, L. Boccardo:

- $H(f(x), u) = u|u|^{r-1} - f(x)$, $f \in L^m(\Omega)$, $r' \leq m < \frac{2N}{N+2} \Longrightarrow u \in W_0^{1,2}(\Omega)$;
- $H(f(x), u) = \frac{u}{u-M} - f(x)$, $0 \leq f \in L^1(\Omega) \Longrightarrow u \in W_0^{1,2}(\Omega)$;
- $H(f(x), u) = -\frac{f(x)}{u(x)}$, $0 \leq f \in L^1(\Omega) \Longrightarrow u \in W_0^{1,2}(\Omega)$;
- $H(f(x), u) = a(x)u - f(x)$, $|f(x)| \leq qa(x) \in L^1(\Omega)$, $q \in \mathbb{R}^+ \Longrightarrow u \in W_0^{1,2}(\Omega) \cap L^\infty(\Omega)$.

If time permits, we will discuss results when the principal part is nonlinear, and H *also depends on* ∇u.

Alice Chang (Princeton University), **Study of the σ_2 functional on closed 4-manifolds**

Integral of σ_2 of the Schouten tensor on closed 4-manifolds is the conformal invariant quantity which is the part of integral of the Chern-Gauss-Bonnet formula module the L^2 of the Weyl curvature part. In the talk, I will first survey some earlier works on the study of the functional; then report some recent work (joint with M. Gursky and Siyi Zhang) where we derive a conformal perturbation result of $\mathbb{C}P^2$ in terms of the size of the functional. I will also report progress made (jointly with Ruobing Zhang and Paul Yang) on the study of the blow up analysis and finite diffeomorphism type of a class of manifolds under the positivity assumption of the functional.

Sagun Chanillo (Rutgers University), **Borderline Sobolev Inequalities after Bourgain-Brezis**

A basic consequence of the 1-variable fundamental theorem of calculus is that for a continuously differentiable function with compact support, one can control the size of the function by the integral of its derivative. Unfortunately this ceases to be true in higher dimensions. One has alternatives like the Moser-Trudinger inequality that plays a fundamental role in Conformal Geometry to the problem of prescribing Gauss curvature. Another alternative to the fundamental theorem of calculus in higher dimensions is the Gagliardo-Nirenberg inequality, which is equivalent to the isoperimetric inequality. In the last decade another inequality has been discovered by Bourgain and Brezis that is a standby for the fundamental theorem of calculus in higher dimensions. We will introduce these inequalities,and then show how the inequality of Bourgain-Brezis extends to the geometric setting of symmetric spaces. Symmetric spaces are manifolds equipped with a Riemannian metric, where for each point one has global isometries that fix the point and reverse geodesics through that point. The Poincare upper half plane, spheres are all examples. Lastly we give applications of the Bourgain-Brezis inequality to the 2-dimensional incompressible Navier-Stokes equation of Fluid Mechanics and the Maxwell equations of Electromagnetism. This is joint work with Po-lam Yung and Jean van Schaftingen.

Marco Degiovanni (Università Cattolica, Brescia), **TBA**

Fanghua Lin (Courant Institute, NYU), **Nodal Sets of Solutions in Homogenization**

It is by now a well-known theory that the control of the geometric measure of nodal sets of solutions to elliptic equations would be dependent of two key properties of solutions. One is the so-called (uniform) doubling condition for the growth, and the other is the uniform C^1 estimate for solutions. The former is often achieved by the Monotonicity of Almgren's frequency function which requires coefficients of the equations to be uniformly Lipschitz. For problems in homogenization, it is not obvious how one can get both of these two key ingredients. In this talk I will sketch *an approach to the problem. The final result is almost identical as for nodal sets of solutions of elliptic equations with smooth coefficients. This is an ongoing joint work with Zhongwei Shen.*

ANDREA MALCHIODI (Scuola Normale Superiore), **Prescribing Gaussian and Geodesic curvature on surfaces with boundary**

We consider the classical problem of finding conformal metrics on a surface such that both the Gaussian and the geodesic curvatures are assigned functions. We use variational methods and blow-up analysis to find existence of solutions under suitable assumptions. A peculiar aspect of the problem is that there are blow-up profiles with infinite volume that have to be taken care of. This is joint work with R. Lopez-Soriano and D. Ruiz.

JIE QING (University California of Santa Cruz), **On Hypersurfaces in Hyperbolic Space**

In this talk I will report our recent works on convex hypersurfaces in hyperbolic space. To study hypersurfaces in hyperbolic space analytically, one needs to find ways to parametrize it, preferably globally. We consider two parametrizations: vertical graph and hyperbolic Gauss map. To get a global parametrization, one needs understand the interrelation of convexity and embeddedness. It is also important to understand the asymptotic of the geometry at ends. In this talk I will report some of our recent works on global and asymptotic properties of hypersurfaces with nonnegative sectional curvature or Ricci curvature in hyperbolic space, where our use of n-Laplace equations seems to be new.

OVIDIU SAVIN (Columbia University), **Quasi Harnack inequality**

We discuss some extensions of the classical Krylov-Safonov Harnack inequality. The novelty is that we consider functions that do not necessarily satisfy an infinitesimal equation but rather exhibit a two-scale behavior. We require that at scale larger than some $r_0 > 0$ *(small) the functions satisfy the comparison principle with a standard family of quadratic polynomials, while at scale* r_0 *they satisfy a Weak Harnack type estimate. We also give several applications of the main result in very different settings such as discrete difference equations, nonlocal equations, homogenization and quasi-minimal surfaces.*

Andrzej Szulkin (Stockholm University), **On a logarithmic Schrödinger equation**

We discuss some recent results on the logarithmic Schrödinger equation

$$-\Delta u + V(x)u = Q(x)u \log u^2, \quad x \in \mathbb{R}^N,$$

where $V, Q > 0$ and Q is bounded. A special feature of this problem is that the Euler-Lagrange functional

$$J(u) := \frac{1}{2}\int_{\mathbb{R}^N}(|\nabla u|^2+(V(x)+Q(x))u^2)\,dx-\frac{1}{2}\int_{\mathbb{R}^N} Q(x)u^2\log u^2\,dx, u \in H^1(\mathbb{R}^N) : \int_{\mathbb{R}^N} V(x)u^2\,dx < \infty$$

is in general only lower semicontinuous and may take the value $+\infty$. In particular, this will happen if V is bounded. On the other hand, if $V \to \infty$ sufficiently fast as $|x| \to \infty$, J takes only finite values. One can show that if V, Q are periodic in $x_1, \ldots, x_N$, then there exist infinitely many geometrically distinct, possibly sign-changing, solutions (Squassina-Sz 2015) and there exist positive multibump solutions (Tanaka-Zhang 2017). We present in some detail the first of these results. The main tool here is a nonsmooth critical point theory. We also briefly discuss the case of nonperiodic V.

Susanna Terracini, **TBA**

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