MATH 71200 - Set Theory and Logic, Spring 2019

Dr. Gunter Fuchs

Wednesday & Friday from 12:15 to 1:45pm, Room 9116

This course is going to develop the concept of a formal proof, or a deduction for first order predicate calculus. This purely syntactic concept will be shown to be equivalent to the semantic notion of logical consequence, which is defined using model theory. This equivalence is known as Gödel's Completeness Theorem, and it will be one of the main results this course is aiming at, the other one being Gödel's Incompleteness Theorem, which says that it cannot be decided by a finitistic algorithm whether a sentence is provable or not. In order to be able to state that theorem precisely, we shall briefly develop the basics of computability theory. All the proofs will take place in the framework of a weak set theory which will be used as the metatheory. This way, this course is intended to show how intricately model theory, computability theory, proof theory and set theory are intertwined.

I will regularly pose homework problems. While I encourage students to discuss the problems together, everybody has to submit their own solutions. Students are asked to use LaTeX to layout their solutions and submit printouts of their work, or to send them by email. There will be a final exam at the end of the semester, offered as part of the logic qualifier. The grade for the course will be the average of the homework grade and the final exam. To succeed in the course, it will be crucial to review the concepts, proofs and ideas presented in class. A great way of doing this is by discussing the topics in small groups. If questions come up, please contact me by email or after class. I will provide notes for the course. These notes will be produced "on the fly" and I will put fairly up-to-date versions of the notes on my home page for download. Additional recommended reading as follows.

References

[End72] Herbert B. Enderton. *A Mathematical Introduction to Logic*. Academic Press, New York, 1972.

[Jec03] Thomas Jech. *Set Theory: The Third Millenium Edition, Revised and Expanded*. Springer Monographs in Mathematics. Springer, Berlin, Heidelberg, 2003.

[Kun80] Kenneth Kunen. *Set Theory. An Introduction To Independence Proofs.* North Holland, 1980.