PROBLEM SET 12 (EXTRA CREDIT) FOR MATH 71200
- SET THEORY AND LOGIC -
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Problem 1:
Let λ be a limit ordinal, with $\kappa = \mathrm{cf}(\lambda)$. Show that there is a cofinal function $f : \kappa \longrightarrow \lambda$ that is strictly increasing and continuous, meaning that if $\alpha < \kappa$ is a limit ordinal, then $f(\alpha) = \bigcup_{\xi<\alpha} f(\xi)$. So f enumerates a club subset of λ.

Problem 2:
Suppose that λ is a singular limit ordinal of uncountable cofinality. Show that there are a stationary subset $S \subseteq \lambda$ and a regressive function $f : S \longrightarrow \lambda$ that is not constant on a stationary subset of S. (So Fodor's Lemma really only holds for uncountable regular cardinals.)

Problem 3:
A train is traveling from 0 to the uncountable regular cardinal κ, making stops at all the intermediate ordinals, in their natural order. It starts out empty, and after its arrival at stop α, the following happens: first, if the train is not empty, then *one* passenger gets off. Second, $\overline{\overline{\alpha}}$ many passengers enter the train. A passenger that has left the train cannot get back on the train (to avoid confusion about whether or not a passenger is on the train or not when it arrives at a limit station).

(1) Show that it is not determined how many passengers will be on the train when it arrives at station ω. In fact, the cardinality of the set of passengers that are on the train when it arrives at station ω could be any number less than or equal to $\aleph_0$.
(2) How many passengers are on the train when it arrives at station κ? Fodor's Lemma is useful here.

Please submit your homework by email, as a pdf file created with LaTeX, by 5/22/2019.