PROBLEM SET 11 FOR MATH 71200 - SET THEORY AND LOGIC - SPRING 2019

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Problem 1:
Fix a finite alphabet Σ. Say that two Turing machines S and T with input alphabet Σ are *equivalent* if for every word w over Σ, S and T exhibit the same behavior. So S accepts w iff T accepts w, and S rejects w iff T rejects w. (Note that it follows that S terminates on input w iff T terminates on input w.) Show that the language

$$E = \{\ulcorner A, B \urcorner \mid A \text{ and } B \text{ are equivalent standard Turing machines}\}$$

is undecidable.
Hint: You can reduce your favorite undecidable problem to E. I.e., show that for a suitable problem P that you know is undecidable, there is a recursive function f such that $f``P \subseteq E$ and $f``P^c \subseteq E^c$. Thus, a decision procedure for E would yield a decision procedure for P.

Problem 2:
Let A be a recursively enumerable set of codes of standard Turing Machines. Show that there is a decidable set B of codes of standard Turing Machines such that modulo equivalence, A is equal to B. I.e., for every $\ulcorner M \urcorner \in A$, there is an $\ulcorner N \urcorner \in B$ such that M and N are equivalent, and vice versa, for every $\ulcorner N \urcorner \in B$, there is an $\ulcorner M \urcorner \in A$ such that M and N are equivalent.

Problem 3:
Use the Recursion Theorem (of Recursion Theory) to show the existence of a Turing machine which outputs its own code (regardless of its input). Put differently, find a natural number n such that φ_n is the constant function with value n.

Please submit your homework by email, as a pdf file created with LaTeX, by 4/28/2019.