PROBLEM SET 10 FOR MATH 71200
- SET THEORY AND LOGIC -
SPRING 2019

DR. GUNTER FUCHS

Problem 1:
Let T be the theory $\mathsf{ZF}^- - \mathsf{Infinity} - \mathsf{Foundation}$. Show that the following formulas are Δ_1^T, or even Σ_0^T:

(1) "$v_1 = \{v_0\}$"
(2) "$v_n = \langle v_0, \dots, v_{n-1}\rangle$."
(3) "v_0 is a function."
(4) "$v_1 = \mathrm{ran}(v_0)$."
(5) "$v_1 = \mathrm{dom}(v_0)$."
(6) "$v_2 = v_0 \restriction v_1$."
(7) "$v_2 = v_0(v_1)$."

Problem 2:
Show that the following functions and relations/sets are $\Delta_1(\mathrm{V}_\omega)$:

(1) ω, and the functions $+$, $\cdot$ defined on ω.
(2) The set Σ of symbols of the language of A_E, in some convenient coding.
(3) The set of variables of that language, also the set of constants, the set of function symbols and the set of relation symbols (the latter three are finite).
(4) The free semi-group generated by these symbols, in the following sense: There is a class $Z \subseteq \mathrm{V}_\omega$ and a function $\frown : Z \times Z \longrightarrow Z$ such that $\langle Z, \frown\rangle$ is a free semi-group generated by Σ, and such that Z and $\frown$ are $\Delta_1(\mathrm{V}_\omega)$.
(5) The set of terms and the set of formulas.
(6) The function π.

Please submit your homework by email, as a pdf file created with LaTeX, by 4/14/2019.