PROBLEM SET 6 FOR MATH 71200 - SET THEORY AND LOGIC - LOGIC I
SPRING 2019

DR. GUNTER FUCHS

Let Δ, Δ' be finite sets of formulas in a fixed language, and let φ, ψ be formulas in that language. Show three of the following properties of the proof calculus introduced in class. In proving property (n), where $n > 1$, you may assume property (m) whenever $1 \leq m < n$.

(1) $\vdash_T \Delta \cup \{\varphi\} \implies \vdash_T \Delta \cup \{\varphi'\}$, where φ' results from renaming a bound variable of φ.
(2) If $\vdash_T \Delta$ and t is a term that does not contain a variable that occurs as a bound variable in a formula from Δ, then $\vdash_T \Delta({}^{x}/_{t})$.
(3) If $\Delta \subseteq \Delta'$ and $\vdash_T \Delta$, then $\vdash_T \Delta'$.
(4) If $\vdash_T \Delta \cup \{(\varphi \vee \psi)\}$, then $\vdash_T \Delta \cup \{\varphi, \psi\}$.
(5) If $\vdash_T \delta \cup \{(\varphi \wedge \psi)\}$, then $\vdash_T \Delta \cup \{\varphi\}$ and $\vdash_T \Delta \cup \{\psi\}$.

Note that when writing $\Delta \cup \{\varphi\}$, it is possible that $\varphi \in \Delta$!

Please submit your homework by email, as a pdf file created with LaTeX, by 3/17/2019.