PROBLEM SET 5 FOR MATH 71200 - SET THEORY AND LOGIC - LOGIC I
SPRING 2019

DR. GUNTER FUCHS

Work in ZF_F^{--}.

Problem 1:
Let Σ be a class. Show that there are terms $\Sigma^*, \frown$ such that $\frown : (\Sigma^*)^2 \longrightarrow \Sigma^*$ and $(\Sigma^*, \frown)$ is a free semi-group generated by Σ.

Problem 2:
Show that there are class terms Var, $F : \omega \rightarrowtail\!\!\!\rightarrow \mathsf{Var}$ and distinct sets $\dot\wedge,\ \dot\vee,\ \dot\neg,\ \dot\forall,\ \dot\exists,\ \dot(,\ \dot)$, $\dot{,},\ \dot=$ such that letting S be the collection of these sets, $\mathsf{Var} \cap S = \emptyset$ and the following holds: if $\mathcal{L} = (\mathbb{C}, \mathbb{P}, \mathbb{F}, \#)$ is a language such that $(\mathbb{C} \cup \mathbb{P} \cup \mathbb{F}) \cap (\mathsf{Var} \cup S) = \emptyset$, then, defining $\Sigma(\mathcal{L})$ as in Def. 1.2.1, there is a class term $\mathsf{Terms}_\mathcal{L}$ that provably satisfies Definition. 1.2.1 (the class of $\mathcal{L}$-terms), using the structure $(\Sigma(\mathcal{L})^*, \frown)$ from Problem 1.

Hint: Use the Recursion Theorem for the "proper substring" relation to define the characteristic function of $\mathsf{Terms}_\mathcal{L}$.

Please submit your homework by email, as a pdf file created with LaTeX, by 3/10/2019.