PROBLEM SET 2 FOR MATH 71200 - SET THEORY AND LOGIC - LOGIC I SPRING 2019

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Problem 1:
Show in ZF_F^{--} that every set is contained in a transitive set, that is, for every a, there is a transitive set u with $a \subseteq u$.

Problem 2:
Let $\mathbb{N} = \{0, 1, 2, \ldots\}$ be the set of natural numbers, and let M^+ and M^- be the models of the language of set theory whose universe is $\mathbb{N}$ and in which $\dot{\in}$ is interpreted as follows:

$$\dot{\in}^{M^+} = < \text{ and } \dot{\in}^{M^-} = >,$$

that is, for $m, n \in \mathbb{N}$, we have that

$$m \dot{\in}^{M^+} n \iff m < n \quad \text{and} \quad m \dot{\in}^{M^-} n \iff m > n.$$

(1) Which axioms of ZF_F^{--} hold in M^+?
(2) Which hold in M^-?
(3) For which $n \in \mathbb{N}$ does $M^+ \models$ "n is transitive"? How about M^-?
(4) If $A = \{x \mid \varphi(x, \vec{y})\}$ is a class term, M is a model for the language of set theory, and $\vec{a}$ is in M, then let's let

$$(A[\vec{a}])^M = \{x \mid \varphi(x, \vec{a})\}^M := \{b \in |M| \mid M \models \varphi(b, \vec{a})\}.$$

That's the relativization of A to M.
 (a) What is $(\{v_0, v_1\}[0, 1])^{M^+}$?
 (b) Show that $M^+ \models \{0, 1\} \in \mathrm{V}$.
 (c) Find the $a \in M^+$ such that $M^+ \models \{0, 1\} = a$.

Please submit your solutions, written in LaTeX or any other layout system you prefer, via email by February 17, 2019.