# CUNY Ph.D Program in Mathematics
# CUNY Graduate Center
Spring 2012

## Hyperbolic 3-manifolds
MATH 86500, 17462
Thursdays: 2 - 4 pm, Room 4433

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**Course Description:** In the 1980s, Bill Thurston revolutionized 3-manifold topology by introducing geometric techniques to study 3-manifolds. Thurston's Geometrization Conjectures, proved by Perelman in 2003, states that 3-manifolds can be cut along spheres and tori in a natural way so that each piece can be given one of the 8 geometric structures. Like in the case of surfaces, hyperbolic geometry is the prevalent and interesting geometry for 3-manifolds. A consequence of Mostow-Prasad rigidity Theorem for hyperbolic structures is that geometric invariants are topological invariants.

In this course we will study complete, finite-volume hyperbolic structures on 3-manifolds, their properties and invariants. We will see numerous classes of examples such as knot complements, surface bundles, arithmetic lattices etc and study computational aspects of hyperbolic 3-manifolds. We will discuss current research topics in hyperbolic 3-manifolds. If time permits, we will study topics relating Kleinian groups, surfaces and 3-manifolds.

**Topics:** We will cover the following topics, roughly in the order below.

1. **Basic 3-manifold theory**: Examples of 3-manifolds, Seifert fibered spaces, Prime Decomposition, JSJ decompositions.
2. **Eight 3-dimensional geometries, Geometrization Theorem**
3. **Basics of $\mathbb{H}^3$:** Geodesics, Isometries, planes, horospheres, hyperbolic volume
4. **Hyperbolic 3-manifolds:** Margulis' Lemma, Mostow-Prasad Rigidity
5. **Geometrization of knot complements and Haken manifolds**
6. **Ideal Triangulations, Angle structures and Examples**
7. **Thurston Dehn Surgery Theorem and Examples**

8. **More examples:** Mapping tori, arithmetic lattices
9. **More topics:** Essential surfaces, commensurability, Arithmetic invariants, deformation of hyperbolic structures.
10. **Computational Aspects: Snappy, Snap**
11. **Topics in Kleinian groups**

**Reference Books:**

1. *Lectures on Hyperbolic Geometry* by Benedetti and Petronio, Springer-Verlag, ISBN 3-540-55534
2. *Low-Dimensional Geometry* by Francis Bonahon, AMS, ISBN 978-0-8218-4816-6
3. *Foundations of Hyperbolic Manifolds* by John G. Ratcliffe, Springer-Verlag GTM 149, ISBN 0-387-94348-x
4. *The Arithmetic of Hyperbolic 3-manifolds* by Machlachlan and Reid, Springer -Verlag GTM 219, ISBN 0-387-98386-4
5. *Thurston's Notes* available at `http://library.msri.org/books/gt3m/`

In addition there will be articles I will use to cover certain topics and assign for reading, references to which I will give during classes.

**Homework and Exams:** Homework problems will be given from time to time. For final project students have to write a two page report using Latex on a recent paper on hyperbolic 3-manifolds which is either uploaded on the arXiv (`http://front.math.ucdavis.edu/math.GT`) or published but not yet reviewed (on MathSciNet). Students should meet with the instructor to choose the paper midway during the semester.