Sample questions for Final Exam

Geometry for Teachers, MTH 623, Fall 2019
Instructor: Abhijit Champanerkar

THE CITY UNIVERSITY OF NEW YORK College of Staten Island

Syllabus for Final Exam:

Sections 5.1-5.3, 6.1 - 6.5 from book, Spherical Geometry, Taxicab Geometry

Definitions

Dividing line segments internally and externally, Transversal of a triangle, Cevians of a triangle, Ideal points and Projective lines, Isometry, Translations, Rotations, Reflections and Glide reflections, 3 reflections theorem, Symmetries of polygons, Spherical distance, Spherical triangle and Polygons, Antipodal map, Taxicab distance, Eulers formula.

Propositions

1. Prop 5.1.3 (pg 181), Prop 5.3.1 (pg 192), Prop 5.3.2 (pg 193)
2. State Ceva's theorem and using Ceva's theorem prove the following:
   (a) The 3 medians are concurrent
   (b) The 3 angle bisectors are concurrent
   (c) The 3 altitudes are concurrent
3. Prop 6.1.2 (pg 201), Prop 6.1.3 (pg 202), Prop 6.2.1 (pg 203), Prop 6.2.2 (pg 205).
4. Let f be any isometry of $\mathbb{R}^2$. Show that f preserves angles.
5. Let f be any isometry of $\mathbb{R}^2$. Show that f takes circles to circles.
6. If a point lies on a great circle, then its antipode also lies on it i.e if $P \in L_{\vec{n}}$ then $-P \in L_{\vec{n}}$. (Proposition 6)
7. Any two distinct great circles intersect in a pair of antipodal points i.e. if $\vec{n_2} \neq \pm \vec{-n}_1$ then $L_{\vec{n_1}} \cap L_{\vec{n_2}} = \{P, -P\}$. (Proposition 8)
8. Any two distinct non-antipodal points on $\mathbb{S}^2$ lie on a unique great circle. (Theorem 9)
9. $|PQ|_{\mathbb{S}^2} = |QP|_{\mathbb{S}^2}$. (Theorem 11)
10. Let P and Q be distinct points on $\mathbb{S}^2$. Then the set of points which are equidistant from P and Q is a great circle.
11. Define the antipodal map and show that it is a spherical isometry.

Problems

In addition to these problems, also look at the problems on homeworks.

1. Let f be an isometry of $\mathbb{R}^2$ given by three reflections, $f = \rho_c \circ \rho_b \circ \rho_a$.
   (a) Suppose the three lines intersect at a single point. Describe the isometry f.

(b) Suppose that a is parallel to b, and c is perpendicular to a. Describe the isometry f.

2. Let f be reflection in the line $x = 1$, and let g be an anti-clockwise rotation of $\pi/4$ about $(0,0)$. Describe $f \circ g$.

3. Give example of two different type of isometries which preserve the $x - axis$.

4. Find translation of $\mathbb{R}^2$ which takes the line $x = y$ to the lines $x = y + 2$.

5. Show that the composition of a rotation by π about $(0,0)$ followed by a rotation by π about $(2,0)$ is a translation. Draw pictures.

6. Find the great circle containing the points $P = (0, 1/2, \sqrt{3}/2)$ and $-P = (0, -1/2, -\sqrt{3}/2)$.

7. Find the distance between the points $P = (1/2, -1/2, 1/\sqrt{2})$, $Q = (2/3, 1/3, -2/3)$.

8. Find angle between the great circles $L_{\langle 1/3, 2/3, 2/3 \rangle}$ and $L_{\langle -3/5, 4/5, 0 \rangle}$.

9. Find the perpendicular bisector for the segments $\overline{PQ}$ where $P = (1/2, -1/2, 1/\sqrt{2})$, $Q = (2/3, 1/3, -2/3)$.

10. Find the sides, angles and area of the triangle with vertices $P = (1,0,0), Q = (0,-1,0)$ and $R = (0,0,-1)$.

11. Can we have a polyhedron consisting of 12 hexagonal faces and every vertex of degree 4 ?

12. Basic calculations in taxicab geometry like distances, circles, equidistant sets etc.