Review for Final Exam

Topology, Math 441, Spring 2020

- **Final will be held on Monday May 18th.** The Final is divided into two parts - (1) An in-class multiple choice/short answer part held online during class time, and (2) a longer take-home part due by midnight on Monday April 18th posted right after class.
- **Syllabus for the take-home Final:** Topics covered from the text book Chapters 5 (connectedness & path-connectedness), Chapter 6 (Compactness) and Surfaces (Euler characteristic, Orientability, Classification etc). The take-home part will have problems like we have seen on the homework.
- **Syllabus for the ~~take-home~~ online Final:** The online part will have multiple choice, true or false, definitions, and examples type of questions covering what we have covered in the whole semester.
- Look at the review sheet for midterm for definitions from material on midterm: `https://www.math.csi.cuny.edu/abhijit/441/resources/review-midterm.pdf`

Definition etc

Connected space
Separation
Connected subspace
Path-connected space
Cutset
Compact space
Compact subset
Bounded set

Convergent sequence
Manifold
Surface
Orientability
connect sum
Triangulation of a surface
Euler characteristic of a surface

Problems from previous in-class Final Exams

1. (a) Define an *open cover* of a subset of a topological space.

   (b) Define a *compact subset* of a topological space.

(c) Let $f : X \to Y$ be a continuous functions. Let $A \subset X$ be compact. Prove that $f(A) \subset Y$ is compact.

(d) Prove that the torus T^2 is compact.

2. (a) Define a *separation* of a topological space.

   (b) Define a *connected* space.

   (c) Let $f : X \to Y$ be a continuous functions. Let X be connected. Prove that $f(X)$ is connected.

   (d) Prove that the torus T^2 is connected.

3. (a) Define a *bounded* subset of $\mathbb{R}^n$.

   (b) Prove that a closed and bounded subset of $\mathbb{R}^n$ is compact.

   (c) Prove that $S^2 \subset \mathbb{R}^3$ is compact.

4. (a) Define a *path connected* space.

   (b) Prove that a path-connected space is connected.

   (c) Prove that $\mathbb{R}^3 - \{\overline{0}\}$ is path connected.

5. (a) Let X_{dis} be X with discrete topology. Prove that if $|X| < \infty$ then X is compact.

   (b) Prove that $\mathbb{R}_\ell$, the lower limit topology of $\mathbb{R}$, is not connected.

   (c) Let $C_1, \ldots C_n$ be compact in X. Prove that $C_1 \cup \ldots \cup C_n$ is compact in X.

6. (a) Let X_{dis} be X with discrete topology. Prove that if $|X| > 2$ then X is disconnected.

   (b) Prove that $\mathbb{R}^n$ is not compact.

   (c) Let $X = \bigcirc$ and $Y = \infty$. Prove that X is not homeomorphic to Y.

7. Identify the following surfaces from the given surface symbols.
   (a) $abca^{-1}b^{-1}c^{-1}$ (b) $abca^{-1}db^{-1}c^{-1}d^{-1}$ (c) $ae^{-1}a^{-1}bddeb^{-1}cc$

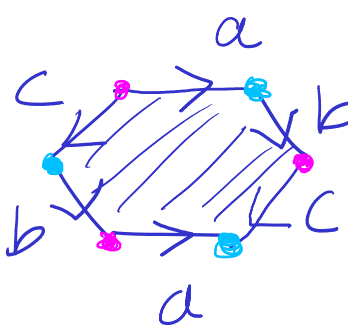
7. Identify surface

(a) $abca^{-1}b^{-1}c^{-1}$ 6 symbols

surface symbol

$\Downarrow$ $\Rightarrow$ hexagon

polygonal representation



$= S$

$v = 2$

$e = 3, \; f = 1$

$$\chi(S) = v - e + f = 2 - 3 + 1 = 0 = 2 - 2g \Rightarrow g = 1$$

nonorint iff $\ldots \underline{a_j} \ldots\ldots \underline{a_j} \ldots$

all symbols appear with inverses

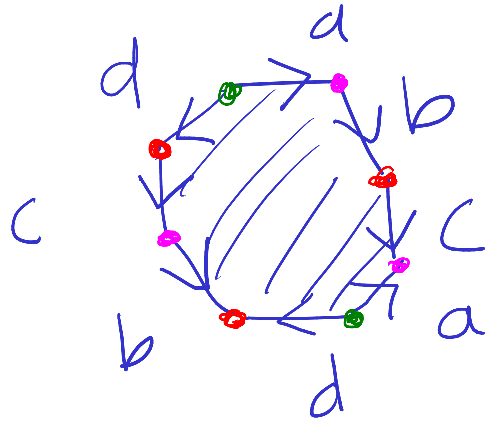
$\Longrightarrow$ orient.

S = orient surface with genus 1

= Torus = T^2

7(b) Identify the surface

$abca^{-1}db^{-1}c^{-1}d^{-1}$ 8 letters $\Rightarrow$ octagon



$= S$

$v = 3$

$e = 4, f = 1$

$\chi(S) = v - e + f = 3 - 4 + 1$
$= 0 = 2 - 2g$
$\Rightarrow g = 1$

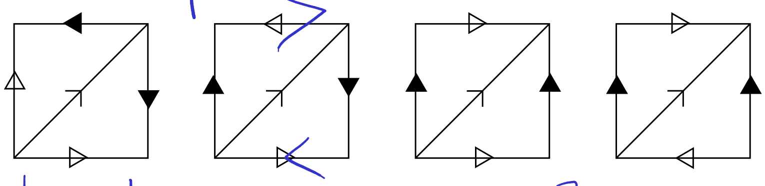
All symbols appear with inverses
$\Rightarrow$ orient.

S = orient surface with genus 1
$= T^2 =$ torus.

8. Identify the surfaces given below.

$aba^{-1}b^{-1}$ $\quad$ $aba^{-1}b$

S^2 $\quad$ P^2

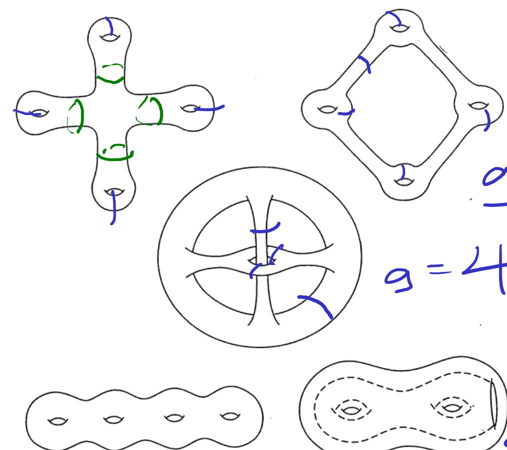


$ab^{-1}ba^{-1}$ $\quad$ $ab^{-1}ab^{-1}$ $\quad$ T^2 $\quad$ K

9. Which of these surfaces are homeomorphic ?

$g = 4$

$g = \#$ cuts on handles to obtain $S^2 - (2g \text{ disks})$



$g = 5$

$g = 4$

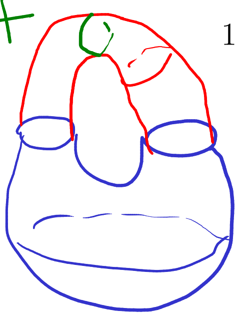
$g = 4$

$g = 4$

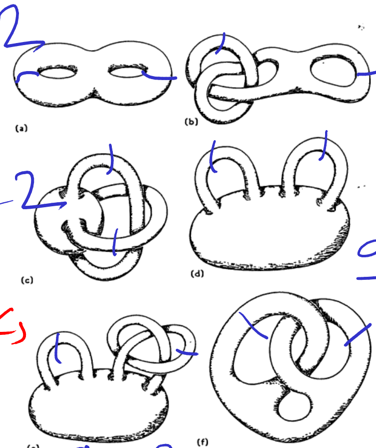
cut along all handles

$= S^2 -$ disks

cut



10. Which of these surfaces are homeomorphic ?



$g = 2$ $\quad$ $g = 2$

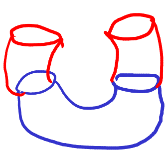
$g = 2$ $\quad$ $= S^2$ with 2 handles

$g = 2$

$g = 2$

$g = 2$

$\Rightarrow$

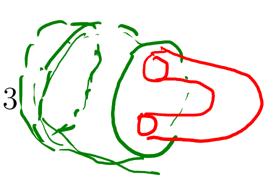


$= S^2 - 2$disks

$\boxed{\text{Handles:}}$

adding cylinders to S^2

$=$



$= T^2 -$ disk