Homework 8

Topology, Math 441, Spring 2020
Topic: Path-connectedness and Compactness
Due: Wednesday April 29th, 2020

THE CITY UNIVERSITY OF NEW YORK College of Staten Island

Reading: pg 77-80, pg 83-89 the text book.

Problems:

1. Prove that the following spaces are path-commnected:
   (a) $\mathbb{R}^2 - \{\overline{x}\}$ (b) $\mathbb{R}^n - \{\overline{x}\}$ is connected for $n \geq 3$, (c) $S^{n-1} \subset \mathbb{R}^n$.

2. Let $X = \{a, b\}$ and assume that X has the topology $\mathcal{T} = \{\phi, \{a\}, X\}$. Prove that X is path connected in this topology.

3. Prove that the following spaces or subsets are non-compact.
   (a) $(a, b) \subset \mathbb{R}$, (b) $[a, b) \subset \mathbb{R}$, (c) $\mathbb{R}^n$

4. Prove that X_{dis} is compact if and only if X is finite.

5. Prove that every subset $K \subset \mathbb{R}_{fc}$ is compact.

6. Suppose that X is a compact space and $(x_n) = \{x_1, x_2, \ldots\}$ is a sequence of points in X. Show that there is a subsequence of (x_n) that converges to a point in X.

7. Prove that if $X \times Y$ is compact then so are X and Y.

8. Prove that a union of two compacts subsets of X is compact.

*9. We have seen that the topologists sine curve is connected. Prove that it is not path-connected.

*10. Let C be a countable subset of $\mathbb{R}^2$. Prove that the $A = \mathbb{R}^2 - C$ is path connected. (Hint: Given a point $\alpha \in A$, there are many lines through α that miss C entirely. Why ?).

Handin: 1a, 3a, 4, 5, 8