Homework 6

Topology, Math 441, Spring 2020
Topic: Subspace and Product Topology
Due: Friday April 3rd, 2020

THE CITY UNIVERSITY OF NEW YORK
College of Staten Island

Reading: Pages 46-56 from Chapter 4.

Problems:

1. Let $Y \subset X$ have the subspace topology.
   (a) Prove that if A is open in Y and Y is open in X then A is open in X.
   (b) Prove that if A is closed in Y and Y is closed in X then A is closed in X.
2. Let $Y = (0, 4] \cup \{5\} \subset \mathbb{R}$. Which of the following subsets of Y are open or closed in the subspace topology on Y ?
   (a) $(0, 1)$ (b) $(0, 1]$ (c) $\{1\}$ (d) $(1, 2)$ (e) $(1, 4]$ (f) $[1, 4]$ (g) $\{5\}$ (h) $\{4, 5\}$
3. Let X be a Hausdorff space and $Y \subset X$. Prove that the subspace topology on Y is Hausdorff.
4. Let $B \subset A \subset X$. Then $x \in A$ is a limit point of B in A if and only if x is a limit point of B in X.
5. Prove that X is Hausdorff if and only if the subset $\Delta(X) = \{(x, x) \mid x \in X\}$ is a closed subset of $X \times X$.
6. Prove that a product of finitely many Hausdorff spaces is Hausdorff (Hint: Prove for two spaces and use induction.)
7. Let $\mathbb{R}_{\text{fc}}$ denote $\mathbb{R}$ with the finite complement topology. Is the finite complement topology on $\mathbb{R}^2$ same as $\mathbb{R}_{\text{fc}} \times \mathbb{R}_{\text{fc}}$?

Handin: 1a, 2, 4, 5