Homework 5

Topology, Math 441, Spring 2020
Topic: Interior, closures etc, and Hausdorff spaces
Due: Monday March 23rd, 2020

Reading: Pages 23-37 from Chapter 3 of the text book. Read subspace topology from Chapter 4.

Problems:

1. Find the interior, closure and boundary of the following sets.
   (a) $\mathbb{Q} \subset \mathbb{R}$, (b) $\mathbb{R} \subset \mathbb{R}^2$, (c) $A = \{(x, y) \mid y \geq 0\} \subset \mathbb{R}^2$.

2. Let X be a topological space and let $A \subset X$. Let ∂A denote the boundary of the set A. Prove the following statements.

   (a) ∂A is a closed set.

   (b) $\partial A = \phi$ if and only if A is both open and closed.

   (c) $\partial A \cap A = \phi$ if and only if A is open.

   (d) $\partial A = \mathrm{Cl}(A) \cap \mathrm{Cl}(X - A)$.

3. (a) Prove that $x \in \mathrm{Cl}(A)$ if and only if every neighbourhood of x interesects A.

   (b) A subset $A \subset X$ is **dense** if $\mathrm{Cl}(A) = X$. Prove $A \subset X$ is dense if and only if for any $x \in X$ and any neighbourhood U of x, $U \cap A \neq \phi$.

4. A space X is **separable** if X has a countable dense set. Prove that a separable metric space is second countable.

5. Prove that $\mathbb{R}_{\mathrm{fc}}$ is not Hausdorff (Hint: Prove that any two (non-empty) open sets in $\mathbb{R}_{\mathrm{fc}}$ intersect).

6. Let X be a Hausdorff space. Prove that any finite set is closed in X.

7. Let (n^2) be a sequence in $\mathbb{R}_{\mathrm{fc}}$. Prove that for any $x \in \mathbb{R}$, $n^2 \to x$.

Handin: 1a, 2ad, 3a, 6, 7