Homework 4

Topology, Math 441, Spring 2020
Topic: Closed sets and limit points
Extended Due Date: Friday March 11th, 2020

Reading: Pages 29-33 from Chapter 3 of the text book.

Problems:

1. For a topological space X with a basis for its topology, we gave a characterization of open sets defined in terms of basis open sets. For any $x \in X$ we define a **neighbourhood of** x to be any open set containing x. Show that a set $U \subset X$ is open if and only if every point $x \in U$ has a neighbourhood contained in U.

2. Prove that the following subsets are closed.

   (a) $\mathbb{Z} \subset \mathbb{R}$.

   (b) $A = \{1/n \mid n \in \mathbb{N}\} \cup \{0\} \subset \mathbb{R}$.

   (c) A closed rectangle $[a, b] \times [c, d] \subset \mathbb{R}^2$ (Hint: use the fact that open rectangles are basis open sets for topology on $\mathbb{R}^2$).

   (d) The $(n-1)$-dimensional sphere $S^{n-1} = \{\, \overline{x} \mid d(\overline{x}, \overline{0}) = 1 \,\} \subset \mathbb{R}^n$.

   (e) $[a, b) \subset \mathbb{R}_\ell$ i.e. $\mathbb{R}$ with lower limit topology.

3. Prove that $\mathbb{Q} \subset \mathbb{R}$ is neither closed nor open.

4. Find the closures of the following sets.

   (a) $(0, 1] \subset \mathbb{R}_\ell$ i.e. $\mathbb{R}$ with lower limit topology.

   (b) $\mathbb{Q} \subset \mathbb{R}$.

5. Let X be a topological space.

   (a) Prove that if $A \subset B$ then $\mathrm{Cl}(A) \subset \mathrm{Cl}(B)$.

   (b) $A \subset X$. Prove that $\mathrm{Cl}(X - A) = X - \mathrm{Int}(A)$.

Handin: 1, 2bce, 3, 4b, 5b