Homework 3

Topology, Math 441, Spring 2020
Topic: Continuos functions and Topological Spaces
Due: Monday March 2nd, 2020

Reading: Pages 19-26 from Chapter 2 of the text book.

Problems:

1. Let (X, d) be a metric space. Show that the following functions are continuous.
   (a) Identity function on X i.e. $f : X \to X$ defined as $f(x) = x$.
   (b) Fix $a \in X$. The function $f_a : X \to \mathbb{R}$ defined by $f_a(x) = d(a, x)$.
2. Let $X = \mathrm{Bdd}([0, 1], \mathbb{R})$ be the metric space of bounded functions with metric defined in the book (see page 16). Let $F : [0, 1] \to \mathbb{R}$ be defined by $F(f) = f(1)$. Show that F is continuous assuming $\mathbb{R}$ has the standard metric.
3. Prove or disprove that the following collection of open sets form a topology on X.
   (a) (Particular point topology) Fix $p \in X$. U is open iff $U = \phi$ or $p \in U$.
   (b) (Excluded point topology) Fix $p \in X$. U is open iff $U = X$ or $p \notin U$.
   (c) $X = \mathbb{R}$. U is open iff $= \phi$ or $U = \mathbb{R}$ or $U = [x, \infty)$ for any $x \in \mathbb{R}$.
4. Let $(X, \mathcal{T}_{dis})$ be the discrete topology on X and let Y be any topological space. Show that every function $f : X \to Y$ is continuous.
5. Let $(Y, \mathcal{T}_{trivial})$ and let X be any topological space. Show that every function $f : X \to Y$ is continuous.

Handin: 1b, 2, 3ab, 4