Homework 1

Topology, Math 441, Spring 2020
Instructor: Abhijit Champanerkar
Due: Monday Feb 10th, 2020

THE CITY UNIVERSITY OF NEW YORK College of Staten Island

Reading:

1. Chapter 1 from the text book.
2. Read and discuss the proof of the Schröder-Bernstein theorem.

Problems:

1. Prove the following:
   (a) $A \times (B \cup C) = (A \times B) \cup (A \times C)$
   (b) $f^{-1}(V - W) = f^{-1}(V) - f^{-1}(W)$
   (c) Prove that $f(A \cap B) \subset f(A) \cap f(B)$. Give an example to show that the converse is not true.
2. Define a relation on $\mathbb{R}$ as $x \sim y$ if and only if $x - y \in \mathbb{Z}$. Show that $\sim$ is an equivalence relation on $\mathbb{R}$. Describe the equivalence class of $1/2$. Describe the space $[\mathbb{R}] = \mathbb{R}/\sim$ i.e. the set of all equivalent classes.
3. Define a relation on $\mathbb{R}^n$ as $\vec{v} \sim \vec{w}$ if and only if $\vec{v} = \lambda \vec{w}$ for a non-zero real number λ. Show that $\sim$ is an equivalence relation on $\mathbb{R}^n$.
4. Let $f : A \to B$ be a function. Define a relation, called *equivalence kernel* on A as: $x \sim y \iff f(x) = f(y)$. Show that this is an equivalence relation. Determine $[\mathbb{R}]$ for the function $f(x) = cos(2\pi x)$.
5. Show that a countable union of countable sets is countable.
6. Show that a finite product of countable sets is countable.
7. Determine, with justification, whether the following sets are countable.
   (a) The set A of all function $f : \mathbb{Z} \to \{0, 1\}$.
   (b) We say $f : \mathbb{N} \to \{0, 1\}$ is eventually zero is there is a positive integer N such that $f(n) = 0$ for all $n \geq N$. The set B of all functions $f : \mathbb{N} \to \{0, 1\}$ which are eventually zero.
   (c) The set C of all finite subsets of $\mathbb{N}$.

Handin: 1bc, 3,4, 5,7bc