Quiz 1

Complex Analysis, MTH 431, Spring 2014
Instructor: Abhijit Champanerkar
Date: Wednesday Feb 5th 2014

Name: SOLUTIONS

1. Write $(1+i)/(3-i)$ in standard form $x + i\,y$.

$$\frac{1+i}{3-i}\,\frac{3+i}{3+i} = \frac{2+4i}{10} = \frac{1}{5} + \frac{2}{5}i$$

2. Find the modulus and principal argument of $z = 2 + i\;2\sqrt{3}$ and use it to show $z^9 = -2^{18}$.

$$r = |z| = \sqrt{4 + 4\times 3} = \sqrt{16} = 4$$

$$\cos\theta = \frac{2}{4} = \frac{1}{2}, \quad \sin\theta = \frac{\sqrt{3}}{2}, \quad \theta = \frac{\pi}{3}$$

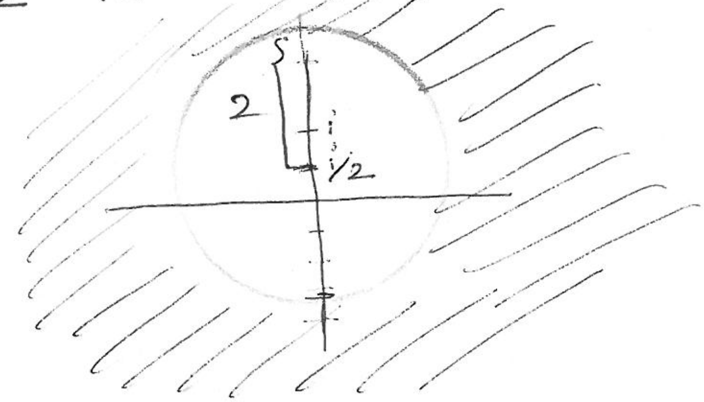
$$z = 4e^{i\pi/3}, \quad z^9 = 4^9 e^{i\frac{\pi}{3}\times 9} = 2^{18} e^{i3\pi} = 2^{18} e^{i\pi}$$

$$= -2^{18}$$

3. Give a geometrical description of the set $\{z \mid |2z - i| \geq 4\}$.

$$|2z - i| \geq 4, \quad |z - i/2| \geq 2$$

$|z - i/2| = 2$ is circle of radius 2, centered at $i/2$



Quiz 2

Complex Analysis, MTH 431, Spring 2014
Instructor: Abhijit Champanerkar
Date: Thursday Feb 20th 2014

Name: SOLUTIONS.

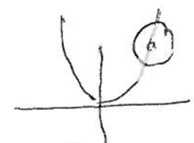
1. Let $f(z) = 2x^3 + iy^2$. Find the set of points on which f is differentiable and f is holomorphic. Determine $f'(z)$ at points where f is differentiable.

4

$u_x = 6x^2 = v_y = 2y$, $y = 3x^2$, u_x, u_y, v_x, v_y cts.

$u_y = 0 = -v_x = 0$

$\Rightarrow$ f diff on parabola $y = 3x^2$



not open set

f not hol. at any pt. $f' = u_x + iv_x = 6x^2$

2. Suppose that $f = u + iv$ is holomorphic. Find v if $u = x^2 - y^2$

3

$u_x = 2x = v_y \Rightarrow v = 2xy + g(x) \Rightarrow v_x = 2x + g'(x)$

$u_y = -2y = -v_x \Rightarrow v_x = 2y$ $\Rightarrow g'(x) = 0$

$g(x) = \text{constant}.$

$\Rightarrow v = 2xy$

3. Show that if f and $\overline{f}$ are both holomorphic on a open, path-connected set U then f is contant on U.

3

$u + iv$ $\quad$ $u - iv$

$u_x = v_y$ $\quad$ $u_x = -v_y$ $\quad \Rightarrow u_x = -u_x \Rightarrow u_x = 0$

$u_y = -v_x$ $\quad$ $u_y = v_x$ $\quad \Rightarrow u_y = -u_y \Rightarrow u_y = 0$

$\Rightarrow u$ constant.
Similarly v constant $\Rightarrow f = \text{constant}.$

Quiz 3

College of Staten Island
The City University of New York

Complex Analysis, MTH 431, Spring 2014
Instructor: Abhijit Champanerkar
Date: Wednesday Feb 26th 2014

Name: SOLUTIONS

$i^0 = 1,\ i^1 = i,\ i^2 = -1,\ i^3 = -i$

Do any **three** of the following. Use formulae given on back.

1. Verify that $e^{iz} = \cos z + i \sin z$.

$$e^{iz} = \sum \frac{i^n z^n}{n!} = \sum_{n=0}^{\infty} (-1)^n \frac{z^{2n}}{(2n)!} + i \sum_{n=0}^{\infty} (-1)^n \frac{z^{2n+1}}{(2n+1)!}$$

$\Rightarrow i^{2n} = (-1)^n$, $i^{2n+1} = i(-1)^n$

$$= \cos z + i \sin z$$

2. Verify that $\cos^2 z + \sin^2 z = 1$.

$$\cos z = \frac{e^{iz} + e^{-iz}}{2}, \quad \sin z = \frac{e^{iz} - e^{-iz}}{2i}$$

$$\cos^2 z + \sin^2 z = \frac{e^{2iz} + e^{-2iz} + 2}{4} + \frac{e^{2iz} + e^{-2iz} - 2}{-4} = 1$$

3. Verify that $e^z = e^x e^{iy} = e^x(\cos y + i \sin y)$ and hence $e^{z+2n\pi i} = e^z$.

From i $\quad e^{iy} = \cos y + i \sin y \Rightarrow e^z = e^x(\cos y + i \sin y)$

$$e^{z+2n\pi i} = e^x(\cos(y+2n\pi) + i \sin(y+2n\pi)) = e^x(\cos y + i \sin y) = e^z$$

4. Verify that $\cosh^2 z - \sinh^2 z = 1$.

$$\cosh z = \frac{e^z + e^{-z}}{2} \quad \sinh z = \frac{e^z - e^{-z}}{2}$$

$$\cosh^2 z - \sinh^2 z = \frac{e^{2z} + e^{-2z} + 2}{4} - \left(\frac{e^{2z} + e^{-2z} - 2}{4}\right) = 1$$