Homework 2

Complex Analysis, MTH 431, Spring 2014

1. Read Remark 3.9 and the discussion about infinity on page 45.

2. page 48-49: 3.4, 3.6, page 61: 4.1ab, 4.3

3. Where are the following functions differentiable? Where are they holomorphic? Determine their derivatives at points where they are differentiable. Use CR equations.

   (a) $f(z) = x^2 + iy^2$

   (b) $f(z) = z^2 - \overline{z}^2$

   (c) $f(z) = z\,\mathrm{Im(z)}$

   (d) $f(z) = |z|^2$

   (e) $f(z) = e^x e^{iy}$

4. Suppose that $f = u + i\,v$ is holomorphic. Find v given u:

   (a) $u = x^2 + y^2$

   (b) $u = x^2 - y^2$

   (c) $u = 2x^2 + x + 1 - 2y^2$

5. Prove the following statements with proper justification. Let U denote an open, path-connected set in $\mathbb{C}$.

   (a) If f is holomorphic on U and always imaginary (i.e. real part equals zero), then f is constant on U.

   (b) If f and $\overline{f}$ are both holomorphic on U then f is contant on U.

   (c) If f is holomorphic on U and range of f lies in either a straight line or a circle, then f is constant. *Hint:* Let the straight line be given by $\mathrm{Re}(aw + b) = 0$ for some $a, b \in \mathbb{C}$. Consider $g(z) := af(z) + b$.

6. Give an example (if they exists) of:

   (a) a non-constant holomorphic function defined on an open set, but has $f' = 0$.

   (b) f is holomorphic such that $|f|$ is constant.

(c) an entire function.

Hand-in Problems **Due:** Wednesday Feb 19th 2014

1. page 48, 3.4.
2. page 61, 4.1b
3. Where are the following functions differentiable? Where are they holomorphic? Determine their derivatives at points where they are differentiable. Use CR equations.
   (a) $f(z) = x^2 + iy^2$
   (b) $f(z) = z^2 - \overline{z}^2$
4. Suppose that $f = u + i\,v$ is holomorphic. Find v if $u = x^2 - y^2$.
5. Prove that if f is holomorphic on U and always imaginary (i.e. real part equals zero), then f is constant on U.