## Sample problems for Exam 1 <br> Introduction to Proof, Math 301, Fall 2023

- Syllabus for Exam 1: Chapters $1 \& 2$ from the book.
- Best way to prepare for the exam is to review definitions and study problems on homework, quizzes and the sample problems below.
- These are only sample problems. The exam will be shorter. We will discuss the format of the Exam during the review.
(1) Find three distinct elements for the truth sets of the following statements:
(a) $x y=1$, where the universe is $\mathbb{R} \times \mathbb{R}$.
(b) $A$ is a subset of $\mathbb{Z}$ which is closed under addition.
(c) $A$ is an element of $\mathcal{P}(\mathbb{Z})$.
(d) $A$ is a subset of $\mathcal{P}(\mathbb{Z})$.
(e) The numbers $a, b$ and $c$ are the lengths of the sides of a right angled triangle.
(2) Consider the statement: If $a \mid c$ and $b \mid c$, then $a b \mid c$.

Which, if any, of the following substitutions give a counter example.
(a) $a=2, \quad b=3, \quad c=12$
(b) $a=3, \quad b=5, \quad c=24$
(c) $a=2, \quad b=2, \quad c=2$
(3) What is $\mathcal{P}(\{\varnothing\})$ ? What is $\mathcal{P}(\mathcal{P}(\{\varnothing\}))$ ?
(4) If $A \cup B \subseteq A \cup C$ does this imply that $B \subseteq C$ ?
(5) State which of the following statements, are true, vacuously true, or false.
(a) If $B \subseteq A \cap B$, then $B \subseteq A$.
(b) If $\mathcal{P}(A)=\varnothing$, then $A=\varnothing$.
(c) If $A \in B$ and $B \in C$, then $A \in C$.
(6) Suppose $A$ and $B$ are finite sets with $|A|=a,|B|=b$ and $|A \cup B|=c$. Find
(a) $|B \backslash A|$
(b) $|A \times(A \cup B)|$
(c) $|\mathcal{P}(A \cap B)|$
(7) Let $A, B$ and $C$ denote sets. Write out a careful proofs of the following:
(a) If $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$.
(b) $A \cap(B \cup C)=(A \cap B) \cup(A \cap C)$.
(c) $(A-B)^{\prime}=B^{\prime}-A^{\prime}$.
(d) If $A \subseteq A \cap B$ then $A \subseteq B$.
(e) If $A \subseteq B$ then $\mathcal{P}(A) \subseteq \mathcal{P}(B)$.
(f) $\mathcal{P}(A \cup B)=\mathcal{P}(A) \cup \mathcal{P}(B)$.
(g) $\mathcal{P}(A \cap B)=\mathcal{P}(A) \cap \mathcal{P}(B)$.
(h) If $A \subseteq B$ and $C \subseteq D$ then $A \times C \subset B \times D$.
(i) If $A \subseteq B$ then $A \times A \subseteq B \times B$.
(j) If $A \times A \subseteq B \times B$ then $A \subseteq B$.
(8) Write out a careful proofs for the following statements about integers. Let $a, b, c$ denote integers.
(a) The product of two odd numbers is odd.
(b) Prove that the difference of squares of odd integers is even.
(c) If $a \mid b$ and $a \mid c$ then $a \mid(4 b-7 c)$.
(d) If $a, b \in \mathbb{N}$ and $a \mid b$ then $a \leq b$.
(9) Write out a careful proofs for the following statements about integers. Let $a, b, c$ denote integers. (Use proof by contradiction).
(a) Prove that if $a b$ is odd then $a$ and $b$ are both odd.
(b) Prove that if $n$ is a natural number and $1 / n^{2}$ is also a natural number than $n=1$.
(c) If $a^{2}$ is even then $a$ is even.
(d) If $a^{2}$ is odd then $a$ is odd.

