Sample problems for Exam 1 Introduction to Proof, Math 301, Fall 2023

- Syllabus for Exam 1: Chapters 1 & 2 from the book.
- Best way to prepare for the exam is to review definitions and study problems on homework, quizzes and the sample problems below.
- These are only sample problems. The exam will be shorter. We will discuss the format of the Exam during the review.
- (1) Find three distinct elements for the truth sets of the following statements:
 - (a) xy = 1, where the universe is $\mathbb{R} \times \mathbb{R}$.
 - (b) A is a subset of \mathbb{Z} which is closed under addition.
 - (c) A is an element of $\mathcal{P}(\mathbb{Z})$.
 - (d) A is a subset of $\mathcal{P}(\mathbb{Z})$.
 - (e) The numbers a, b and c are the lengths of the sides of a right angled triangle.
- (2) Consider the statement: If a|c and b|c, then ab|c.

Which, if any, of the following substitutions give a counter example.

(a) a = 2, b = 3, c = 12(b) a = 3, b = 5, c = 24(c) a = 2, b = 2, c = 2

- (3) What is $\mathcal{P}(\{\emptyset\})$? What is $\mathcal{P}(\mathcal{P}(\{\emptyset\}))$?
- (4) If $A \cup B \subseteq A \cup C$ does this imply that $B \subseteq C$?
- (5) State which of the following statements, are true, vacuously true, or false.
 (a) If B ⊂ A ∩ B, then B ⊂ A.
 - (a) If $\mathcal{P}(A) = \emptyset$, then $A = \emptyset$.
 - (c) If $A \in B$ and $B \in C$, then $A \in C$.
- (6) Suppose A and B are finite sets with |A| = a, |B| = b and $|A \cup B| = c$. Find (a) $|B \setminus A|$
 - (b) $|A \times (A \cup B)|$
 - (c) $|\mathcal{P}(A \cap B)|$
- (7) Let A,B and C denote sets. Write out a careful proofs of the following:
 (a) If A ⊆ B and B ⊆ C then A ⊆ C.

- (b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.
- (c) (A B)' = B' A'.
- (d) If $A \subseteq A \cap B$ then $A \subseteq B$.
- (e) If $A \subseteq B$ then $\mathcal{P}(A) \subseteq \mathcal{P}(B)$.
- (f) $\mathcal{P}(A \cup B) = \mathcal{P}(A) \cup \mathcal{P}(B).$
- (g) $\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B).$
- (h) If $A \subseteq B$ and $C \subseteq D$ then $A \times C \subset B \times D$.
- (i) If $A \subseteq B$ then $A \times A \subseteq B \times B$.
- (j) If $A \times A \subseteq B \times B$ then $A \subseteq B$.
- (8) Write out a careful proofs for the following statements about integers. Let a, b, c denote integers.
 - (a) The product of two odd numbers is odd.
 - (b) Prove that the difference of squares of odd integers is even.
 - (c) If a|b and a|c then a|(4b 7c).
 - (d) If $a, b \in \mathbb{N}$ and a|b then $a \leq b$.
- (9) Write out a careful proofs for the following statements about integers. Let a, b, c denote integers. (Use proof by contradiction).
 - (a) Prove that if ab is odd then a and b are both odd.
 - (b) Prove that if n is a natural number and $1/n^2$ is also a natural number than n = 1.
 - (c) If a^2 is even then a is even.
 - (d) If a^2 is odd then a is odd.