Homework 8 Introduction Mathematical Proof, MTH 301, Fall 2023 Instructor: Abhijit Champanerkar Points: 30 Due: Monday Dec 11th, 2023



**Reading:** Its really important in this class that you read the book carefully, working through the Explorations and GYHD's as you go.

Sections 5.1 and 5.2.

## Homework Problems (from text book):

- 1. Section 5.1, Pages 196-198: 3ab, 9, 11, 12ab, 14ab, 17
- 2. Problems on the Induction Practice Problem sheet below.

Handin Problems: These problems are to be handed in class. Write up clear solutions to the following problems.

- 1. Section 5.1, Pages 196-198: 11, 17a
- 2. From "Induction Practice Problems": 1d, 1h, 1r, 1u

## **Induction Practice Problems**

- 1. Prove the following statements using either the First or Second Principle of Mathematical Induction. Be sure to state somewhere in your proof (probably best in the conclusion) which principle you used.
  - (a) For all  $n \in \mathbb{N}$ ,  $1 + 3 + 5 + 7 + (2n 1) = n^2$
  - (b) For all  $n \in \mathbb{N}$ ,  $1 + 3 + 3^2 + \ldots + 3^{n-1} = \frac{3^n 1}{2}$
  - (c) For all  $n \in \mathbb{N}$ ,  $1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4}$
  - (d) For all  $n \in \mathbb{N}$ ,  $\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}$
  - (e) For all  $n \in \mathbb{N}$ ,  $1 \cdot 3 + 2 \cdot 3^2 + 3 \cdot 3^3 + \ldots + n \cdot 3^n = \frac{(2n-1)3^{n+1}+3}{4}$
  - (f) For all  $n \in \mathbb{N}$ ,  $1 + 2 + 2^2 + \dots + 2^{n-1} = 2^n 1$ .
  - (g) Prove that n! > 3n for all  $n \in \mathbb{N}_{[7,\infty)}$
  - (h) Prove that  $2n < n^2$  for all  $n \in \mathbb{N}_{[4,\infty)}$
  - (i) Show that  $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \dots + \frac{1}{\sqrt{n}} \le 2\sqrt{n}$  for
  - (j) Show that  $2! + 4! + 6! + \dots + (2n)! \le ((n+1)!)^n$  for all  $n \in \mathbb{N}$ .
  - (k) Any convex polygon with n sides can be cut into n-2 triangles. Prove it.
  - (l) Prove that  $3^n < n!$  for all  $n \in \mathbb{N}_{[6,\infty)}$ .
  - (m) Prove that n(n+1)(n+2) is divisible by 6 for all  $n \in \mathbb{N}$ .
  - (n) Prove that  $1 + 4 + 7 + ... + (3n 2) = \frac{1}{2}n(3n 1)$  for all  $n \in \mathbb{N}$ .
  - (o) Prove that  $10^n + 3 \cdot 4^{n+2} + 5$  is divisible by 9 for all  $n \in \mathbb{N}$ .
  - (p) Prove that  $(2n+7) < (n+3)^2$  for all  $n \in \mathbb{N}$ .
  - (q) Prove that  $1 + 2 + 3 + ... + n < \frac{1}{8}(2n+1)^2$  for all  $n \in \mathbb{N}$ .
  - (r) Prove that  $10^{2n-1} + 1$  is divisible by 11 for all  $n \in \mathbb{N}$ .
  - (s) Prove that  $x^{2n} y^{2n}$  is divisible by x + y for all  $n \in \mathbb{N}$ .
  - (t) Prove that  $3^{2n+2} 8n 9$  is divisible by 8 for all  $n \in \mathbb{N}$ .
  - (u) Prove that the number of all the subsets of a set containing n distinct elements is  $2^n$  for all  $n \in \mathbb{N}$ .
  - (v) Prove that the product of two consecutive natural numbers is an even number.

- (w) Prove that 7+77+777+...+777...7 (n digits)  $=\frac{7}{81}(10^{n+1}-9n-10)$ for all  $n \in \mathbb{N}$ .
- (x) Let  $F_k$  be the Fibonacci numbers defined by  $F_1 = 1, F_2 = 1$ , and for k > 2,  $F_k = F_{k-1} + F_{k-2}$ . Show that the following formulas hold for all  $n \in \mathbb{N}$ :
  - (a)  $F_{n-1}F_{n+1} = F_n^2 + (-1)^n$ (b)  $\sum_{i=1}^n F_i^2 = F_n F_{n+1}$ (c)  $\sum_{k=1}^n F_k = F_{n+2} 1$

  - (d)  $F_1 + F_3 + F_5 + \dots + F_{2n+1} = F_{2n+2}$ .
- (y) Consider the following four equations:

$$1 = 1$$
 (1)

2+3+4 = 1+8(2)

$$5 + 6 + 7 + 8 + 9 = 8 + 27 \tag{3}$$

$$10 + 11 + 12 + 13 + 14 + 15 + 16 = 27 + 64 \tag{4}$$

Conjecture the general formula suggested by these four equations, and prove your conjecture.