Sample Problems for Exam 3

Course : Introduction to Probability and Statistics, Math 113 Section 3234
Instructor: Abhijit Champanerkar

- Exam 3 will be held in class on Wednesday Nov 28th.

- Syllabus for Exam 3: Sections 4.7, 5.1, 5.2, 5.3, 5.4, 6.2, 6.3, 6.6

- Review for Exam 3 will be held on Monday Nov 26th.

- Best way to prepare for the midterm is to solve the Sample problems and MyStatLab problems.

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1. **Permutations and Combinations**

   (a) How many ways can you arrange 3 apples, 4 oranges, 2 plums and 4 pears in a line ?

   (b) How many ways can you form a team of 3 men and 3 women from a class of 12 men and 10 women ?

   (c) How many four digit PINs can you make if the first digit cannot be a 0 ? A digit is a number between 0 to 9.

   (d) How many ways can you arrange the letters of the word "MAGIC" ?

2. **Probability Distributions, Mean and Variance**

   (a) The following probability distribution gives the probability of buses breaking down on the route from CSI to the Ferry terminal.

Number of buses x	2	3	4	5	6	7	8	9
Probability $p(x)$	0.10	0.23	0.22	0.19	0.14	0.08	0.03	0.01

      i. What is the probability that exactly 5 buses are late ? exactly 8 buses are late ?

      ii. Find $p(x \geq 4)$.

      iii. What is the probability that at least 4 buses are late ?

      iv. What is the probability that at most 4 buses are late ?

      v. Find $p(2 \leq x \leq 6)$.

      vi. How many buses do you expect to be late (compute the expected value i.e. mean) ?

      vii. What is the standard deviation of this distribution ?

   (b) Consider the following card game. A single card is dealt from a standard deck of 52. The following values are given to the cards: if the card is a NOT a face card, it is worth 1 point, if the card is a face card (Jack, Queen, King) it is worth 2 points, if the card is an Ace, it is worth 3 points.

    i. Identify, in words, the random variable X in the game and write down its range (all possible values of X).

    ii. Find the probability distribution of X and sketch it (relative histogram).

    iii. Find the expected value (mean) of X.

3. **Binomial Distribution**

  (a) At the end of the Slosson Avenue exit ramp of the Expressway there is a traffic light that is Green 80% of the time for traffic exiting.

    i. What is the probability that a car exiting on 7 different occasions never sees a red light?

    ii. What is the probability that a car exiting on 7 different occasions sees exactly five times a red light?

  (b) Adam is taking a multiple choice exam that consists of 10 questions. Each question has 5 possible answers. Adam guesses at every answer. What is the probability that he passes the exam if he needs at least 5 correst answers to pass the exam ?

4. **Normal distribution (Draw pictures to solve the problems)**

  (a) In a standard normal distribution, find the area that lies

    i. between $z = -1.42$ and $z = 0.86$.

    ii. between $z = 1.24$ and $z = 2.73$.

    iii. to the right of $z = -0.42$.

    iv. to the left of $z = -0.53$.

  (b) In a normal distribution with $\mu = 40$ and $\sigma = 7$, find the percentage of scores that are

    i. between 35 and 48.

    ii. greater than 39.

  (c) A soft drink company produces 330 ml bottles of cola. The volume of cola in each bottle is approximately normally distributed with mean 330 ml and standard deviation 8 ml. Calculate the probability that a bottle will contain less than 310 ml of cola.

  (d) Waiting time at a toll plaza for a bridge for cars paying cash during off-peak hours is normally distributed with a mean of 8.7 minutes and standard deviation of 1.7 minutes (imagine during peak hours !). If a commuter at the toll plaza is randomly selected, what is the probability that a commuter will have to wait

    i. between 6 and 9 minutes.

    ii. more than 10 minutes.

  (e) In a standard normal distribution, find P_{75} (75-th percentile).

5. **Normal curve approximation to Binomial distribution (use correction factor)**

  (a) OnTime airlines finds that 18% of all reservations are subsequently changed. If 200 reservations are randomly selected, what is the probability that between 30 and 40 of them will subsequently be changed.

  (b) If 12% of the population is left-handed, estimate the probability that there are at least 20 left-handers is a school of 200 students.