Graphical Solutions to Equations

Project 4– Exercises

NAME: __________________________________________
SECTION: __________________________________________
INSTRUCTOR: __________________________________________

Exercise 1:
Use a graph of 
\[(x - 2)^2 = 4 \sin(x)\]
to find solutions to the equation valid to 2 decimal points:

(1) Answer: __________________________

Exercise 2:
Use the zooming technique to find solutions of

\[50 + \sin(x) = 2x\]

which are valid to at least two decimal places.

*Hint: Try to estimate the value of 50 + \sin(x). This will give you an idea in which x interval are the possible solutions!*

(2) Answer: __________________________

Exercise 3:
Folklore is that exponential functions grow faster than polynomial functions. Although true, you need to be careful about how you interpret this statement, as this exercise shows.

Consider the functions \(z_1 = e^x\) and \(z_2 = x^4\). Plot them together on the interval \([0,4]\).

a. From their graphs, how can you determine which graph is the exponential and which is the polynomial?

(3) Circle one:
1. polynomial functions grow faster than exponential functions
2. Exponential functions grow faster than polynomial functions
3. For different values of \(x\), I can evaluate \(z_1, z_2\) and determine which is larger.
b. Find the value of \( x \) (to two decimal places) for the point of intersection by zooming on the zero of \( f(x) = e^x - x^4 \). (or by zooming on the intersection point of the functions \( z_1 = e^x \), \( z_2 = x^4 \).)

(4) Answer: 

On this graph, \( x^4 \) is larger than \( e^x \) from the intersection point to \( x = 4 \). Experiment to determine how large a value of \( x \) is needed for the exponential to catch up to \( x^4 \). Then find the second intersection point. (correct to three decimal places.) This one is larger than 4. In fact, you now have found two intersection points \( (x_1, y_1), (x_2, y_2) \). (where \( x_1 < x_2 \)) Up to \( x_1 \) the function \( e^x \) is bigger, from \( x_1 \) to \( x_2 \) the function \( x^4 \) is the bigger. What happens after \( x_2 \)?

c. What is the \( x \)-coordinate of the second intersection point?

(5) Answer: 

d. What happens to the behavior of \( z_1 \) and \( z_2 \) after the second intersection point?

(6) Circle one:
1. \( e^x \) grows faster
2. \( x^4 \) grows faster
3. they grow at the same rate
4. \( e^x \) grows faster, but for increasingly large values of \( x \), \( x^4 \) catches up to \( e^x \) again.

Exercise 4:

a. Find the \( x \)-coordinate for where \( f(x) = (x + 2)/x^2 \) achieves its minimum value.

(7) Answer: 

b. What interval on the \( x \)-axis did you use to make you plot window?

(8) Answer:
Exercise 5:

a. Let \( f(x) = x^3 - 7x^2 + 2x + 9 \). Solve the cubic equation \( f(x) = 0 \). Find all of its roots correctly up to 4 significant digits.

(9) Circle one:
1. 6.6, 1.1, -0.7
2. 6.4766, 1.4692, -0.9458
3. 6.7053, 1.3259, -0.8259
4. 0.0010, 1.0100, 7.5902
5. 6.5806, 1.1062, -0.6868

b. Now find all solutions to \( x^3 + 2x + 4 = 0 \) (Note that the coefficient of \( x^2 \) is now 0).

(10) Circle one:
1. 0.6641, -0.6640, -1.3283
2. 1.8230, -1.8230, -1.3283
3. 0.5898 ± 1.7445i, -1.1795
4. 1.8230 ± 0.6641i, -1.3283

Exercise 6:

a. Let \( \theta = \pi/4 \). Look carefully at \( f(x) \), it is a quadratic polynomial in \( x \). Rewrite \( f(x) \) so that the coefficients appear as (careful with the scientific notation)

\[
f(x) = ax^2 + bx + c.
\]

Now represent this polynomial in MATLAB, as in \([a \ b \ c]\). What are the values:

(11) Answer:
b. Use your previous answer and the `roots` function to find the range \([0, b]\) of an arrow when shot at an angle of \(\pi/4\). Specify the range in terms of its endpoint \(b\).

(12) Answer: ________________________________

Exercise 7:

a. Plot various graphs of the \(g(x)\) until you find the range of \(g, [0, b]\). Enter the value of \(b\) with at least 1 digit to the right of the decimal point. (Remember, arrows don’t bounce up – this mathematical model is only valid until the arrow first hits the ground.)

(13) Answer: ________________________________

b. Now make a plot containing the trajectories of both models. Label the individual plots.

(14) Attach your graph to the worksheet.

c. From your graph estimate the maximum height of the arrow if there is no wind resistance.

(15) Answer: ________________________________

d. From your graph estimate the maximum height of the arrow if there is wind resistance.

(16) Answer: ________________________________