MTH 700 Topology I, Fall 12, HW3

(1) Let T be the torus $S^1 \times S^1$. Show that if x is a point in T, then $T \setminus \{x\}$ deformation retracts to the wedge of two circles $S^1 \vee S^1$, also known as the figure eight.

(2) Let x be a point in S^1. Show that there is a retraction from $T = S^1 \times S^1$ to $S^1 \times \{x\}$, but no deformation retraction.

(3) For a path connected space X, show that $\pi_1(X)$ is abelian if and only if all change of basepoint homorphisms β_h depend only on the endpoints of the path h.

(4) Let A, B and C be compact subsets of $\mathbb{R}^3$. Use Borsuk-Ulam to show that there is a plane P which simultaneously divides each of the sets into two parts of equal measure. (This is the ham sandwich theorem, i.e. given a ham sandwich (two slices of bread and some ham), you can cut it precisely in half in one go.)

(5) Show that every homomorphism from $\pi_1(S^1)$ to $\pi_1(S^1)$ can be realized as the induced homorphism of a map from S^1 to S^1.

(6) Hatcher p39 Q16.

(7) Hatcher p39 Q17. Remember to show that the maps you construct are really not homotopic.