9a) Columns are basis $\Rightarrow$ linearly independent columns

$\Rightarrow$ Nullspace$(A) = 0$ since there are no relations between cols.

b) Columns basis $\Rightarrow$ Columns span $\mathbb{R}^5$ $\Rightarrow$ every $\vec{b} \in \mathbb{R}^5$ can be written as linear combination of columns $\Rightarrow$ $A\vec{x} = b$ has solution.

c) Full column rank $\Rightarrow$ rank$(A) = 5$ $\Rightarrow$ Full row rank

$\Rightarrow$ rows of A are linearly independent $\Rightarrow$ 5 of them, so they span.

10a) $\left.\begin{array}{l}\text{One basis vector}\\ \text{in } C(A)\end{array}\right\}$ rank$(A)=1$ $\Rightarrow$ $\dim(N(A)) = 1 = n - r = n - 1$

$\Rightarrow n = 2$ so every vector in $N(A) \subset \mathbb{R}^2$, not $\mathbb{R}^3$.

(Another way to say this: $r + (n-r) = n$, here $1 + 1 = 2 \Rightarrow n = 2$)

10 b) Sorry, this is possible! $\begin{bmatrix} 1 & 1 \\ 0 & 2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 1 \\ 2 & 4 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ works.

10 c) $A\vec{v} = 0 \Rightarrow$ (every row of A) $\cdot\, \vec{v} = 0 \Rightarrow \vec{v} \cdot \vec{v} = 0$

But $\vec{v} \cdot \vec{v} = 1 + 0 + 1 = 2 \neq 0$.

10 d) $A^T y = 0 \Rightarrow y = 0$ means that $\dim$(left nullspace) $= 0 = m - r$

$\Rightarrow r = m$ (A has full row rank)

$\Rightarrow \{A\vec{x}\}$ spans $\mathbb{R}^m$, so every $\vec{b} \in \mathbb{R}^m$ in image of A

$\Rightarrow A\vec{x} = \vec{b}$ has solutions for all $\vec{b} \in \mathbb{R}^m$.