Calculus III (Math 233) Quiz 2

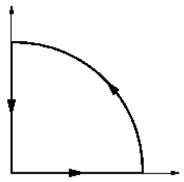
Date: November 21, 2016 Professor Ilya Kofman

NAME: ______________________________________

Problem 1. Find the volume of the solid enclosed by $z = x^2 + y^2$ and $z = 8 - x^2 - y^2$.

Problem 2. Find the volume of the solid enclosed between the double-cone $z^2 = x^2 + y^2$ and the sphere $x^2 + y^2 + z^2 = 9$. (This solid includes the xy–plane inside the sphere.)

Problem 3. Compute $\int_C y\,dx - x\,dy$, where C is the path around the quarter-circle of radius 3 as shown:



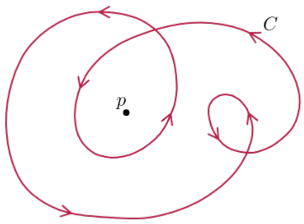
Problem 4. Consider the vector field $\mathbf{F} = \langle\, 2e^z,\, 2y,\, 2xe^z \,\rangle$. Let C be any curve from $(1, 0, 1)$ to $(1, 1, 0)$.

(a) Show that $\mathbf{F}$ is conservative (without using part (b)).

(b) Find the potential function for $\mathbf{F}$.

(c) Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$.

Problem 5. Let $\mathbf{F} = \left\langle \dfrac{-y}{x^2 + y^2}, \dfrac{x}{x^2 + y^2} \right\rangle$. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r} =$ ____________ where C is the path around the origin $p = (0, 0)$ as shown:



Problem 6. Let S be the surface $z = x\sqrt{3} + y^2$, for $-1 \le x \le 1$ and $0 \le y \le 1$. Evaluate $\iint_S x^2 y\, dS$.

Problem 7. Let S be the upper hemisphere: $x^2 + y^2 + z^2 = 1,\ z \ge 0$. Let C be the boundary of S in the xy–plane. Do NOT compute the following integrals.

(a) Explain why $\int_C xyz\, ds = 0$. (b) Explain why $\iint_S xyz\, dS = 0$.