Exam 1 Math 233 9/28/2016 Solutions

① a) $\ell(t) = P + t\vec{v}$, $\vec{v} = \vec{R} - \vec{P} = (-1,-2,2) \Rightarrow \ell(t) = (1,1,0) + t(-1,-2,2)$
$= (1-t,\ 1-2t,\ 2t)$

b) $\vec{PQ} = (-3,0,0)$, $\vec{PR} = (-1,-2,2)$

$\vec{n} = PQ \times PR = \begin{vmatrix} i & j & k \\ -3 & 0 & 0 \\ -1 & -2 & 2 \end{vmatrix} = \vec{i}(0) - \vec{j}(-6) + \vec{k}(6) = (0,6,6)$

$\vec{n}\cdot(\vec{x} - P) = 0 \Rightarrow (0,6,6)\cdot(x-1, y-1, z) = 0 \Rightarrow \underline{y + z = 1}$

c) $\text{Area}_{\Delta PQR} = \frac{1}{2}\|PQ \times PR\| = \frac{1}{2}\sqrt{36+36} = \frac{6\sqrt{2}}{2} = 3\sqrt{2}$

d) $y + z = 1 \Rightarrow -t + (4+t) = 1 \Rightarrow 4 = 1 \Rightarrow \Leftarrow$ Do <u>not</u> intersect

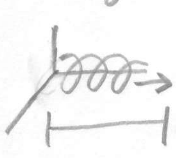
---

② Speed $v(t) = \|\vec{r}\,'(t)\|$, $\vec{r}\,'(t) = (-12\sin 3t,\ 5,\ 12\cos 3t)$

a) $= \sqrt{144\sin^2(3t) + 25 + 144\cos^2(3t)} = \sqrt{144+25} = 13$

b) $\vec{T}(t) = \frac{1}{13}\vec{r}\,'(t) = \left(-\frac{12}{13}\sin 3t,\ \frac{5}{13},\ \frac{12}{13}\cos 3t\right)$

c) $s = \int_0^{2\pi} \|r'(t)\|\,dt = \int_0^{2\pi} 13\,dt = (13)(2\pi) = 26\pi$

d) 

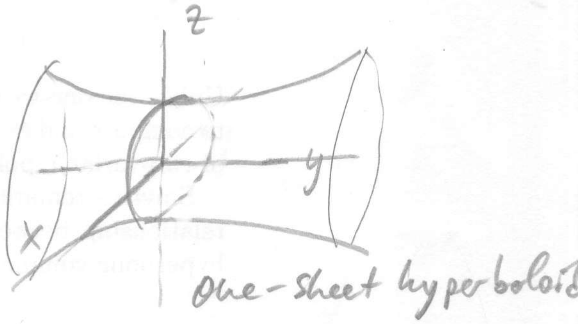
helix twists in <u>positive y-direction</u>, <u>height 10π</u> above circle of <u>radius 4 in xz-plane</u> making <u>3 full revolutions</u> over circle

---

③ $x = 0 \Rightarrow z^2 - 3y^2 = 13$

a) $y = 0 \Rightarrow x^2 + z^2 = 13$

$z = 0 \Rightarrow x^2 - 3y^2 = 13$



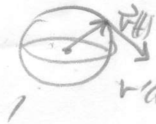
One-sheet hyperboloid

b) $F(x,y,z) = x^2 - 3y^2 + z^2$ $\quad -4x - 6y + 3z = 13$

$\nabla F(x,y,z) = (2x, -6y, 2z) \Rightarrow \nabla F(-4,2,3) = (-8,-12,6) \Rightarrow \vec{n} = (-4,-6,3)$

$\vec{n}\cdot(\vec{x} - P) = 0 \Rightarrow (-4,-6,3)\cdot(x+4, y-2, z-3) = 0 \Rightarrow -4(x+4) - 6(y-2) + 3(z-3) = 0$

④ $\|\vec{r}(t)\| = R$ where R = radius of Earth $\Rightarrow \|\vec{r}(t)\|^2 = R^2$



$\vec{r}\cdot\vec{r} = R^2 \Rightarrow \frac{d}{dt}(\vec{r}\cdot\vec{r}) = 0 = \vec{r}\cdot\vec{r}\,' + \vec{r}\,'\cdot\vec{r} = 2\vec{r}\cdot\vec{r}\,'$

$\Rightarrow \vec{r}(t)\cdot\vec{r}\,'(t) = 0$, so velocity $\vec{r}\,'(t) \perp$ position $\vec{r}(t)$, hence tangen

---

⑤ a) Let $x=0$. $\lim\limits_{(0,y)\to(0,0)} \frac{xy^2}{x^2+y^4} = \lim\limits_{y\to 0} \frac{0}{y^4} = 0.$

Let $x=y^2$ $\lim\limits_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^4} = \lim\limits_{y\to 0} \frac{y^4}{2y^4} = \frac{1}{2}$

$\Big\}$ Limit DNE.

b) $g_x = \frac{1}{2}(5xy+2z)^{-1/2}(5y)$ $\quad g_y = \frac{1}{2}(5xy+2z)^{-1/2}(5x)$ $\quad g_z = \frac{1}{2}(5xy+2z)^{-1/2}(2)$

c) $h_x = \frac{1}{z^2+1}(2x)$ $\quad h_y = \frac{1}{z^2+1}(2y)$ $\quad h_z = (x^2+y^2)\left(-\frac{2z}{(1+z^2)^2}\right)$

$h_{xz} = -\frac{(2z)(2x)}{(1+z^2)^2}$ $\quad h_{xy} = 0$ $\quad h_{yz} = -\frac{(2y)(2z)}{(1+z^2)^2}$

---

⑥ a) $f_x = \frac{4x}{2x^2-6y^2}\Big|_{(2,1)} = \frac{8}{8-6} = 4$ $\quad f_y = \frac{-12y}{2x^2-6y^2}\Big|_{(2,1)} = \frac{-12}{8-6} = -6$

$f(2,1) = \log(8-6) = \log 2 \Rightarrow z = \log 2 + 4(x-2) - 6(y-1)$

$\underline{4x - 6y - z = 2 - \log 2}$

b) $F(x,y,z) = xy - yz + zx$

$\nabla F = (y+z,\ x-z,\ x-y)\big|_{(2,0,3)} = (3,-1,2) = \vec{n}$

$\vec{n}\cdot(\vec{x} - (2,0,3)) = 0 \Rightarrow (3,-1,2)\cdot(x-2,\ y,\ z-3) = 0$

$3(x-2) - y + 2(z-3) = 0 \Rightarrow \underline{3x - y + 2z = 12}$

---

⑦ $f_x(P) = 3,\ f_y(P) = -4 \to \nabla f(P) = (3,-4)$ = answer for (a)

b) $f(1,-2) = z\big|_P = 3(1) - 4(-2) + 7 = 18$

$f(1.02,-2.01) \approx 18 + f_x(P)(0.02) + f_y(P)(-0.01)$

$= 18 + (3)(0.02) + (-4)(-0.01)$

$= 18 + 0.06 + 0.04$

$= 18.1$

(8) a) $\nabla T = (-12x^2, -6y)\big|_{(-1,1)} = (-12,-6)$

b) $D_{(3,4)}T = \nabla T(-1,1) \cdot \dfrac{(3,4)}{\|(3,4)\|} = (-12,-6) \cdot \dfrac{(3,4)}{5} = \frac{1}{5}(-60) = -12$

c) $\nabla T(-1,1) = (-12,-6)$

d) $\|\nabla T(-1,1)\| = \sqrt{144+36} = 6\sqrt{5}$

e) Any $\vec{u}$ such that $\vec{u} \cdot (-12,-6) = 0$

If $\vec{u} = (a,b)$ then $2a + b = 0$ eg. $(1,-2)$ or $(-1,2)$