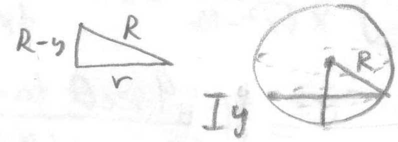
Exam 1 Math 232 3/9/2015 Solutions

① $r^2 = R^2 - (R-y)^2, \quad 0 \le y \le H$

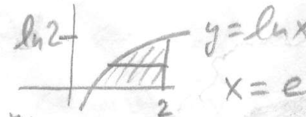


$V = \int_0^H A(y)\,dy = \int_0^H \pi r^2\,dy = \pi \int_0^H R^2 - (R-y)^2\,dy$

$= \pi \int_0^H 2Ry - y^2\,dy = \pi \left[Ry^2 - \frac{1}{3}y^3 \right|_0^H = \underline{\underline{\pi\left(RH^2 - \frac{1}{3}H^3\right)}}$

② $V = \int_0^3 A(y)\,dy = \int_0^3 \frac{\pi r^2}{2}\,dy$ where $2r = x = 3 - y$

$= \frac{\pi}{2}\int_0^3 \left(\frac{1}{2}(3-y)\right)^2 dy = \frac{\pi}{8}\int_0^3 (3-y)^2\,dy = \frac{\pi}{8}\left(-\frac{1}{3}(3-y)^3\right|_0^3 = \underline{\underline{\frac{9\pi}{8}}}$



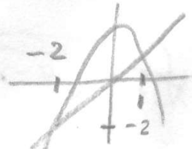
③ $V = \int_0^{\ln 2} 2\pi r h\,dy = 2\pi \int_0^{\ln 2} y(2 - e^y)\,dy = 2\pi \int_0^{\ln 2} 2y - ye^y\,dy$

$u = y \quad dv = e^y dy$

$du = dy \quad v = e^y$

$= 2\pi y^2\Big|_0^{\ln 2} - 2\pi\left[ye^y - e^y\right|_0^{\ln 2}$

$= 2\pi(\ln 2)^2 - 2\pi[(\ln 2 - 1)2 + 1] = \underline{\underline{2\pi\left((\ln 2)^2 - 2\ln 2 + 1\right)}}$



④ (shell) $V = 2\pi \int_{-2}^1 rh\,dx = \underline{\underline{2\pi \int_{-2}^1 (1-x)(2-x^2-x)\,dx}}$

⑤ (washer) $V = \pi \int_{-2}^1 (R^2 - r^2)\,dx = \underline{\underline{\pi \int_{-2}^1 (2-x^2+2)^2 - (x+2)^2\,dx}}$

⑥ $\int_0^2 x^2 e^{3x}\,dx = \left[\frac{1}{3}x^2e^{3x} - \int \frac{2}{3}xe^{3x}\right|_0^2 = \left[\frac{1}{3}x^2e^{3x} - \frac{2}{3}\left[\frac{1}{3}xe^{3x} - \frac{1}{3}\int e^{3x}dx\right|_0^2\right.$

$u = x^2 \quad dv = e^{3x}dx$

$du = 2x \quad v = \frac{1}{3}e^{3x}$

$u = x \quad dv = e^{3x}dx$

$du = dx \quad v = \frac{1}{3}e^{3x}$

$= \left[\frac{1}{3}x^2e^{3x} - \frac{2}{9}xe^{3x} + \frac{2}{27}e^{3x}\right|_0^2$

$= \frac{4}{3}e^6 - \frac{4}{9}e^6 + \frac{2}{27}e^6 - \frac{2}{27} = \underline{\underline{\frac{2}{27}(13e^6 - 1)}}$

⑦ $\int \sin^2(12x)\,dx = \frac{1}{2}\int 1 - \cos(24x)\,dx = \underline{\underline{\frac{1}{2}x - \frac{1}{48}\sin(24x) + C}}$

⑧ $\int \cos^3(6x)\sin^8(6x)\,dx = \int (1-\sin^2(6x))\sin^8(6x)\cos(6x)\,dx = \int (1-u^2)u^8\left(\frac{1}{6}du\right)$

$u = \sin(6x), \ du = 6\cos(6x)$

$= \underline{\underline{\frac{1}{6}\left[\frac{1}{9}\sin^9(6x) - \frac{1}{11}\sin^{11}(6x)\right] + C}}$

⑨ $\int \sqrt{9-5x^2}\,dx = \int \sqrt{9(1-\frac{5}{9}x^2)}\,dx = 3\int \sqrt{1-\frac{5}{9}x^2}\,dx = 3\cdot\frac{3}{\sqrt{5}}\int \cos^2\theta\,d\theta$

$\begin{cases} x = \frac{3}{\sqrt{5}}\sin\theta \\ dx = (3/\sqrt{5})\cos\theta\,d\theta \end{cases}$

$+C$

$= \frac{9}{\sqrt{5}}\cdot\frac{1}{2}\left(\theta + \frac{1}{2}\sin(2\theta)\right) = \frac{9}{2\sqrt{5}}\left(\theta + \sin\theta\cos\theta\right) = \frac{9}{2\sqrt{5}}\left(\sin^{-1}\left(\frac{\sqrt{5}x}{3}\right) + \frac{\sqrt{5}x}{3}\cdot\sqrt{1-\frac{5x^2}{9}}\right)$

$= \frac{9}{2\sqrt{5}}\sin^{-1}\left(\frac{x\sqrt{5}}{3}\right) + \frac{x}{2}\sqrt{9-5x^2} + C$

(10) $\int \frac{1}{x\sqrt{x^2-16}}\,dx \qquad x = 4\sec\theta \qquad \sqrt{x^2-16} = \sqrt{16(\sec^2\theta - 1)} = 4\tan\theta$

$dx = 4\sec\theta\tan\theta\,d\theta$

$$= \int \frac{4\sec\theta\tan\theta\,d\theta}{(4\sec\theta)(4\tan\theta)} = \frac{1}{4}\int d\theta = \frac{1}{4}\theta + C = \underline{\underline{\frac{1}{4}\cos^{-1}\left(\frac{4}{x}\right) + C}}$$

(11) $\frac{x^2-21x+50}{(x-2)(x^2-4)} = \frac{x^2-21x+50}{(x-2)^2(x+2)} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{x+2}$

$A(x-2)(x+2) + B(x+2) + C(x-2)^2 = x^2-21x+50$

$x=2 \Rightarrow 4B = 12 \Rightarrow B = 3$

$x=-2 \Rightarrow 16C = 96 \Rightarrow C = 6$

x^2- term $\Rightarrow A + C = 1 \Rightarrow A = -5$

$$\int \frac{-5}{x-2} + \frac{3}{(x-2)^2} + \frac{6}{x+2}\,dx =$$

$$= \underline{\underline{-5\ln|x-2| - \frac{3}{x-2} + 6\ln|x+2| + C}}$$

(12) $\int \frac{4x^4+100x^2+7}{x^2+25}\,dx = \int 4x^2 + \frac{7}{x^2+25}\,dx = \underline{\underline{\frac{4}{3}x^3 + \frac{7}{5}\tan^{-1}\left(\frac{x}{5}\right) + C}}$

(13) $\int_0^2 x\sqrt{5-\sqrt{4-x^2}}\,dx \qquad u = 4-x^2$

$du = -2x\,dx$

$$= \int_4^0 -\frac{1}{2}\sqrt{5-\sqrt{u}}\,du = \frac{1}{2}\int_0^4 \sqrt{5-\sqrt{u}}\,du$$

$v = 5-\sqrt{u}$

$dv = -\frac{1}{2\sqrt{u}}\,du \Rightarrow du = -2\sqrt{u}\,dv$

$\Rightarrow du = -2(5-v)\,dv$

$$= \frac{1}{2}\int_5^3 \sqrt{v}\,(-2(5-v))\,dv$$

$$= -\int_5^3 5\sqrt{v} - v^{3/2}\,dv = \left[-\frac{10}{3}v^{3/2} + \frac{2}{5}v^{5/2}\right]_5^3$$

$$= \left(-\frac{10}{3}3^{3/2} + \frac{2}{5}3^{5/2}\right) - \left(-\frac{10}{3}5^{3/2} + \frac{2}{5}5^{5/2}\right) = \frac{20\sqrt{5}}{3} - \frac{32\sqrt{3}}{5} \approx 3.822$$