MTH/SLS 218–6816 Exam 3

May 4, 2011 Professor Ilya Kofman

NAME: Key

3pts each (21)

Problem 1. The following statistics provide information about a national biology exam, taken by 8,000 students:

Mean 429, Median 365, Mode 371
25th Percentile 150, 75th Percentile 583, 90th Percentile 764

(a) 4,000 students scored at or above 365 points.

(b) 2,000 students scored at or above 583 points.

(c) The number of students who scored 764 or <u>below</u> was 7200.

(d) The number of students who scored 150–764 was 5200.

(e) Among the best 4,000 students, their median was 583 points.

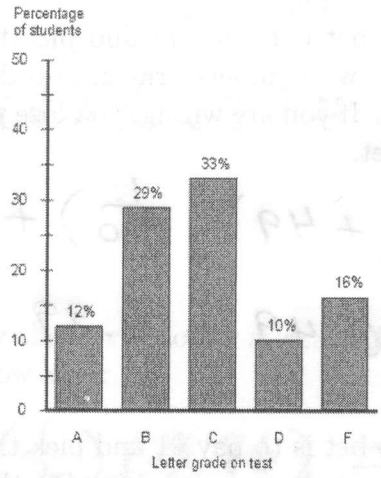
(f) Together, all 8,000 students earned 3,432,000 points in total.

(g) Estimate the percentile for the most common score: $\approx$ 60th

accept any $50 \leq X < 75$

3pts each (15)

Problem 2. The following are the results of a test given to 500 students. Test grades are recorded as $A = 4$, $B = 3$, $C = 2$, $D = 1$, $F = 0$ points.



(a) The median on this test is 2 points.

(b) The mean on this test is 2.11 points.

(c) Students in the 75th percentile earned 3 points.

(d) Students who earned a B are in what percentile? 59th

(e) Together, all 500 students earned 1055 points in total.

108

36

Problem 3. Scores on a recent SAT were roughly normal, with mean 1057 points, and standard deviation 226 points.

4 (a) What was the range of the middle 99.7% of SAT scores?

$379 - 1735$

4 (b) Frank scored in the 16th percentile on the SAT. What was his score?

831

4 (c) What percentage of students scored at or above 605?

97.5%

6 (d) An older SAT had mean 1043 and standard deviation 234 points. If Jack scored 1072 on the recent SAT, and Jill scored 1061 on the older SAT, who scored better? Show work.

Jack $z = \frac{1072-1057}{226} = 0.066$

Jill $z = (1061-1043)/234 = 0.077$ (Better)

4 pts each

Problem 4. In the Minnesota lottery *Daily-3* game, the digits 0-9 are used, and 3 of these digits are selected one at a time (repetition is allowed). Ways to play:

(a) The first way to bet is to pay \$1 and pick a single digit. If your digit matches the first number drawn, then you win \$5 (your \$1 bet plus \$4 more). If you are wrong, you lose your dollar bet. Find the expected value of this bet.

off by \$1 -2

(12)

$EV = (+4)(\frac{1}{10}) + (-1)(\frac{9}{10})$

$= 0.4 - 0.9 = -0.5$

(b) Another way to bet is to pay \$1 and pick two digits, in order. If your digits match the first two numbers drawn, in order, then you win \$50 (your \$1 bet plus \$49 more). If you are wrong, you lose your dollar bet. Find the expected value of this bet.

$EV = (+49)(\frac{1}{100}) + (-1)(\frac{99}{100})$

$= 0.49 - 0.99 = -0.5$

(c) The final way to bet is to pay \$1 and pick three digits, in order. If your digits match all three numbers drawn, in order, then you win \$500 (your \$1 bet plus \$499 more). If you are wrong, you lose your dollar bet. Find the expected value of this bet.

$EV = (+499)(\frac{1}{1000}) + (-1)(\frac{999}{1000})$

$= 0.499 - 0.999 = -0.5$

6

Problem 5. (a) A box contains 6 red marbles and 4 black marbles. Two marbles are drawn without replacement from the box. What is the probability that both marbles are black?

$$\frac{4 \cdot 3}{10 \cdot 9} = \frac{2}{15}$$

6

(b) You roll a fair 6-sided die labelled 1–6, and a fair 4-sided die labelled 1–4. To win, you must roll 1,3 or 3,1. What is the probability of winning?

$$\frac{1}{6} \cdot \frac{1}{4} + \frac{1}{6} \cdot \frac{1}{4} = \frac{2}{24} = \frac{1}{12}$$

6 pts each

Problem 6. The probability that a random person was born on a Monday is 14%. In a randomly selected family, suppose that boys and girls are equally likely.

(a) What is the probability that two kids were both born on Mondays?

$\frac{1}{7} \cdot \frac{1}{7} = \frac{1}{49}$ OR $(0.14)(0.14) = 0.0196$

(b) What is the probability that neither of two kids was born on Mondays?

$\frac{6}{7} \cdot \frac{6}{7} = \frac{36}{49}$ OR $(0.86)(0.86) = 0.7396$

(c) If there are three kids, what is the probability that there are two older boys and the youngest is a girl?

B B G $\quad (\frac{1}{2})(\frac{1}{2})(\frac{1}{2}) = \frac{1}{8}$

(d) What is the probability that a randomly selected child is a boy who was NOT born on Monday? Show work.

$\frac{1}{2} \cdot \frac{6}{7} = \frac{3}{7}$ OR $(\frac{1}{2})(0.86) = 0.43$

(e) **Bonus:** What is the probability that a randomly selected child is either a boy or else NOT born on Monday? Show work.

P(Boy OR (Girl AND NOT Monday)) $= \frac{1}{2} + (\frac{1}{2})(0.86)$

$\frac{1}{2} + \frac{3}{7} = \frac{13}{14}$ OR $= 0.5 + 0.43 = 0.93$