

March 28, 2011

Professor Ilya Kofman

NAME: Key

1 in = 2.54 cm, 1 kg = 2.2 lbs 1 gal = 3.79 l

20 **Problem 1.** A recipe that serves 6 calls for 2 Tbsp vegetable oil, 4 pints of stock, $\frac{2}{3}$ lbs of peas, and 5 eggs. Note that 16 Tbsp = 1 cup. If the cafeteria needs to serve 1000, it must buy:

(a) How many gallons of vegetable oil? 1.3 gal

$$\frac{1000}{6} \cdot 2 \text{ Tbs} \cdot \frac{1 \text{ cup}}{16 \text{ Tbsp}} \cdot \frac{1 \text{ gallon}}{16 \text{ cups}} = \frac{2000}{1536} \text{ gal}$$

(b) How many liters of stock? 315.8 l

$$\frac{1000}{6} \cdot 4 \text{ pts} \cdot \frac{1 \text{ gal}}{8 \text{ pts}} \cdot \frac{3.79 \text{ l}}{\text{gal}} = \frac{(4000)(3.79)}{48} \text{ l} = \boxed{315.8 \text{ l}}$$

(c) How many kilograms of peas? _____

$$\frac{1000}{6} \cdot \frac{2}{3} \text{ lbs} \cdot \frac{1 \text{ kg}}{2.2 \text{ lbs}} = \boxed{50.5 \text{ kg}}$$

(d) How many dozens of eggs? _____

$$\frac{1000}{6} \cdot 5 \text{ eggs} \cdot \frac{1 \text{ dozen}}{12 \text{ eggs}} = 69.4 \Rightarrow \boxed{70 \text{ dozen eggs}}$$

10 **Problem 2.** Standard copy paper is 8.5 inches by 11 inches.

(a) How many square millimeters (mm^2) is one sheet of copy paper?

$$(8.5)(11) \text{ in}^2 \times \left(\frac{2.54 \text{ cm}}{1 \text{ in}}\right)^2 \times \left(\frac{10 \text{ mm}}{1 \text{ cm}}\right)^2 = 60,322 \text{ mm}^2$$

(b) One acre is 43,560 sq ft. How many pieces of copy paper will cover one acre?

$$43,560 \text{ ft}^2 \times \left(\frac{12 \text{ in}}{1 \text{ ft}}\right)^2 \times \frac{1 \text{ paper}}{(8.5)(11) \text{ in}^2} = 67,087 \text{ papers}$$

20 Problem 3. (a) How many km is 72,531 mm?

$$0.072531 \text{ km}$$

(b) How many km is 5.4 miles?

$$5.4 \text{ mi} \times \frac{5280 \text{ ft.}}{1 \text{ mi}} \times \frac{12 \text{ in}}{1 \text{ ft.}} \times \frac{2.54 \text{ cm}}{1 \text{ in}} \times \frac{1 \text{ km}}{100,000 \text{ cm}} = 8.7 \text{ km}$$

(c) How fast is 3 cm/sec in miles per hour?

$$3 \frac{\text{cm}}{\text{sec}} \times \frac{1 \text{ in}}{2.54 \text{ cm}} \times \frac{1 \text{ ft}}{12 \text{ in}} \times \frac{1 \text{ mile}}{5280 \text{ ft.}} \times \frac{60 \text{ sec}}{1 \text{ min}} \times \frac{60 \text{ min}}{1 \text{ hr.}} = 0.067 \frac{\text{miles}}{\text{hr.}}$$

(d) The current price of gasoline in France is 1.58 Euros per liter. Now, \$1 is 0.71 Euros. How much is gasoline in France in dollars per gallon?

$$1.58 \frac{\text{Euro}}{\text{l}} \times \frac{\$1}{0.71 \text{ Euro}} \times \frac{3.79 \text{ l}}{1 \text{ gal}} = \$8.43/\text{gal}$$

5 Problem 4. Jack reports that a pail weighs 4.7 lbs. Jill can weigh things in pounds to three decimal places. If Jill weighs the pail, what weight range could she report?

$$4.650 - 4.749$$

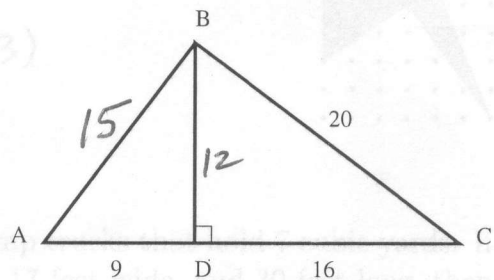
15 Problem 5. In $\triangle ABC$, $\angle BDC$ is a right angle, but do not assume that $\angle ABC$ is a right angle.

(a) Find the area of $\triangle ABC$.

$$16^2 + (BD)^2 = 20^2$$

$$BD = \sqrt{20^2 - 16^2} = 12$$

$$\text{Area}(\triangle ABC) = \frac{1}{2} (12)(9+16) = 150$$



(b) Find the perimeter of $\triangle ABC$.

$$9^2 + (BD)^2 = (AB)^2 \Rightarrow 9^2 + 12^2 = (AB)^2 \Rightarrow AB = 15$$

$$\text{Perimeter}(\triangle ABC) = 15 + 20 + 25 = 60$$

(c) Determine whether $\angle ABC$ is a right angle. Justify.

Yes, since $(AB)^2 + (20)^2 = (25)^2$ (Pythagorean Thm.)
 $15^2 + 20^2 = 25^2$

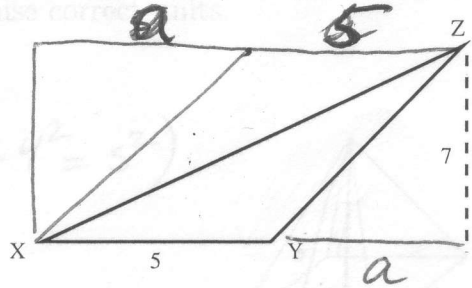
6

Problem 6. Justify $A = \frac{1}{2}bh$ for $\triangle XYZ$, using rectangles and/or right triangles.

$$7(a+5) = 2(7a)\left(\frac{1}{2}\right) + 2 \text{ Area}(\triangle XYZ)$$

$$\Rightarrow 7 \cdot 5 = 2 \text{ Area}(\triangle XYZ)$$

$$\Rightarrow \text{Area}(\triangle XYZ) = \frac{1}{2}(7)(5)$$



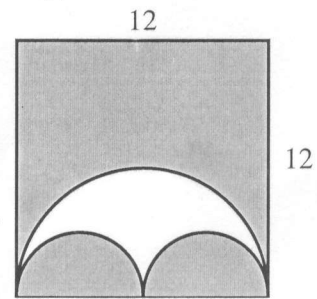
6

Problem 7. Find the area of the shaded region. (Leave π in your answer.)

$$\text{Area} = (12)(12) - \frac{1}{2}\pi(6)^2 + \pi(3)^2$$

$$= 144 - \frac{1}{2}\pi(6)^2 + \pi(3)^2$$

$$= 144 - 9\pi$$



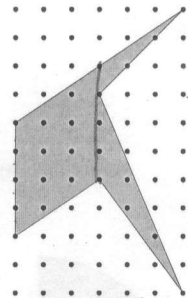
6

Problem 8. The dots below are spaced 1 cm apart. Determine the area of the shaded figure. Show work.

$$A = \square + \triangle + \triangle$$

$$= (3)(4) + \frac{1}{2}(1)(3) + \frac{1}{2}(3)(3)$$

$$= 18$$



6

Problem 9. A construction company has dump trucks that hold 7 cubic yards. If the company digs a hole that is 12 feet deep, 17 feet wide, and 30 feet long, then how many dump trucks will they need to haul away the dirt dug from this hole?

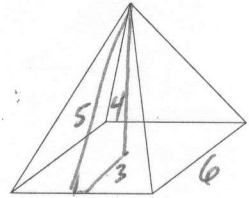
$$\text{Vol (hole)} = \underbrace{12 \cdot 17 \cdot 30}_{6120} \text{ ft}^3 \times \left(\frac{1 \text{ yd}}{3 \text{ ft}}\right)^3 = 226.7 \text{ yd}^3$$

$$\frac{226.7}{7} = 32.4 \Rightarrow \underline{33 \text{ dump trucks}}$$

20 **Problem 10.** A square with side length 6 cm is the base of a square pyramid, which has height 4 cm. Show work below, and use correct units.

(a) What is the slant height of the pyramid?

$$\underline{s = 5 \text{ cm}} \quad (3^2 + 4^2 = s^2)$$



(b) Compute the surface area of the pyramid (including the base).

$$4 \cdot \frac{1}{2} (6)(5) + (6)^2 = 96 \text{ cm}^2$$

(c) Compute the volume of the pyramid.

$$\frac{1}{3} (36) (4) = 48 \text{ cm}^3$$

(d) If a little cube with side length 5 mm is filled with water, how many such cubes will fill the pyramid?

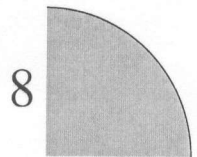
$$\text{Each cube vol} = \left(\frac{1}{2} \text{ cm}\right)^3 = \frac{1}{8} \text{ cm}^3$$

$$\# \text{ cubes} = \frac{(48) \text{ cm}^3}{\left(\frac{1}{8} \text{ cm}^3\right)} = 384 \text{ cubes}$$

15 **Problem 11. (BONUS)** A cone will be made from the quarter-disc shown.

(a) Find the surface area of the cone.

$$SA = \frac{1}{4} \pi (8)^2 = 16\pi$$



(b) Find the radius r of the cone. (Hint: Use circumference.)

$$2\pi r = \frac{1}{4} (2\pi \cdot 8) \Rightarrow r = 2$$



(c) Find the volume of the cone. (Hint: Find the height.)

$$\left. \begin{array}{l} r^2 + h^2 = s^2 \\ 2^2 + h^2 = 8^2 \end{array} \right\} h = \sqrt{64 - 4} = \sqrt{60}$$

$$V = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi (2^2) \sqrt{60} = \frac{4\pi \sqrt{60}}{3}$$