MTH/SLS 218–6816 Exam 3

May 12, 2008 Professor Ilya Kofman

NAME: Key

Problem 1. A right rectangular prism is $1\frac{1}{2}$ cm long, $\frac{2}{3}$ cm wide, $\frac{3}{4}$ cm deep. Express your answers as fractions in lowest terms, and use correct units.

(a) What is the volume of the prism?

6 $\frac{3}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} = \frac{3}{4}\,cm^3$

4/6 for LHS

(b) What is the surface area of the prism?

Base Ph

OR $2(\frac{3}{2}\cdot\frac{2}{3}) + (2\cdot\frac{3}{2} + 2\cdot\frac{2}{3})\cdot\frac{3}{4}$

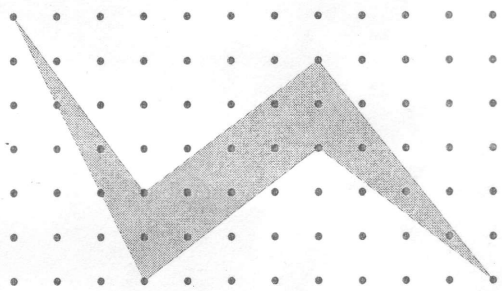
6 $2\cdot\frac{3}{2}\cdot\frac{2}{3} + 2\cdot\frac{3}{2}\cdot\frac{3}{4} + 2\cdot\frac{2}{3}\cdot\frac{3}{4}$

$= 2 + \frac{9}{4} + 1 = 5\frac{1}{4} = \frac{21}{4}\,cm^2$

Problem 2. The dots below are spaced 1 cm apart. Determine the area of the shaded figure. Show work.

6 $\frac{1}{2}\cdot 2\cdot 3 + 2\cdot 4 + \frac{1}{2}\cdot 2\cdot 4$

$= 3 + 8 + 4 = 15\,cm^2$

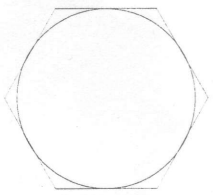


Problem 3. A regular hexagon with side length $\sqrt{3}$ can be circumscribed about a circle with diameter 3.

(a) The area of the hexagon is $\frac{9\sqrt{3}}{2}$. Compute the area of the circle. What fraction of the hexagon's area is covered by the circle? (Leave your answer in terms of $\sqrt{3}$ and π.)

3 $r = \frac{3}{2}$ $A_\bigcirc = \pi(\frac{3}{2})^2 = \frac{9\pi}{4}$

2 pts for πr^2 with $r = 1.5$ ≈ 7.07



3 $\frac{A_\bigcirc}{A_\hexagon} = \frac{9\pi/4}{9\sqrt{3}/2} = \frac{\pi}{2\sqrt{3}}$

(b) Using the hexagon and circumference, find an upper bound for π. (i.e., Find a number b such that $\pi \le b$.)

3 $C = 2\pi(\frac{3}{2}) = 3\pi < P_\hexagon = 6\sqrt{3} \Rightarrow \underline{3\pi < 6\sqrt{3}}$

3 $\pi < 2\sqrt{3}$

30

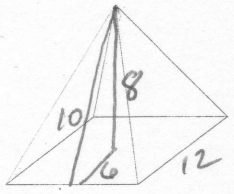
Problem 4. A square with side length 12 cm is the base of a square pyramid, which has height 8 cm. Show work below, and use correct units.

(a) What is the slant height of the pyramid?

5

$s^2 = 6^2 + 8^2$

$s = 10$ cm



(b) Compute the surface area of the pyramid.

5

$SA = 12^2 + \frac{1}{2}P \cdot s = 144 + \frac{1}{2} \cdot 48 \cdot 10$

$= 144 + 240 = 384$ cm^2

(c) Compute the volume of the pyramid.

5

$V = \frac{1}{3} \cdot 12^2 \cdot 8 = 384$ cm^3

(d) **Bonus:** If a cone has the same volume and height as the pyramid, find the radius of the cone. (Leave your answer in terms of π.)

5

$384 = \frac{1}{3}\pi r^2 h = \frac{8}{3}\pi r^2 = \frac{8}{3} \cdot 12^2$

$r^2 = \frac{12^2}{\pi} \Rightarrow r = \frac{12}{\sqrt{\pi}}$

Problem 5. The following ratings were recorded at a dog show:

23, 41, 46, 49, 55 | 58, 60, 65, 71, 73 | 75, 75, 78, 79, 80 | 85, 85, 88, 94 | 96

50th 75th 95th

(a) Find the median rating.

3

74

(b) If dogs ~~in~~ at or above the 75th percentile went to the next round, what were their ratings?

5

(except 80 – 96) ~~dead dead~~ 85–96

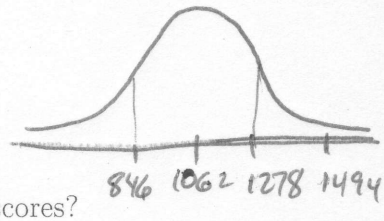
(c) The top dog is in which percentile at this show?

5

95th percentile (top dog is in top 5%)

28 + 5

Problem 6. Scores on a recent SAT were roughly normal, with mean 1062 points, and standard deviation 216 points.



(a) What was the range of the middle 68% of SAT scores?

3 $846 - 1278$

(b) How high must a student score to be in the top 2.5% of SAT scores?

5 1494

$1494 - 1170$ 3pts.

(c) What is the probability that a randomly selected student scored above 846?

5 $84\% = 0.84$

(d) **Bonus:** An older SAT had mean 1075 and standard deviation 202 points. If Jack scored 1170 on the recent SAT, and Jill scored 1173 on the older SAT, who scored better? Show work.

5 Jack $\frac{1170-1062}{216} = 0.5$ $>$ Jill $\frac{1173-1075}{202} = 0.49$

Problem 7. Two spinners each have three equal regions. Spinner 1 is marked $\{1, 3, 5\}$. Spinner 2 is marked $\{2, 4, 6\}$. The game is to spin each spinner once, and add up the scores. Hint: Make a chart of possible outcomes.

	1	3	5
2	3	5	7
4	5	7	9
6	7	9	11

(a) What is the probability that your total is even?

3 0

(b) What is the probability that your total is not 9?

5 $7/9$

(c) What is the expected value of this game?

5 $\frac{1}{9}(3+5+5+7+7+7+9+9+11) = 7$

Problem 8. As above, Spinner 1 is marked $\{1, 3, 5\}$. This game is to spin the spinner and toss a coin that many times.

(a) What is the probability that you spin 1 and get H?

5 $\frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6}$

(b) What is the probability that you spin 3 and get HHH?

5 $\frac{1}{3} \cdot \frac{1}{8} = \frac{1}{24}$

1/8 2pts.

(c) **Bonus:** What is the probability that you play this game and get all heads on the tosses?

5 $\frac{1}{6} + \frac{1}{24} + \frac{1}{3} \cdot \frac{1}{32} = \frac{16+4+1}{96} = \frac{21}{96} = \frac{7}{32}$

36 + 10