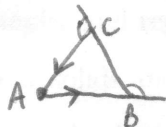


February 27, 2008

Professor Ilya Kofman

NAME: Key

- 6 **Problem 1.** Consider an equilateral  $\triangle ABC$ . A person begins walking straight at point  $A$ , turns at  $B$  and then at  $C$ , and returns to  $A$  but does not turn. What is the total number of degrees that the walker has turned? Justify your answer.



$$120 + 120 = 240$$

- 6 **Problem 2. (a)** If a pyramid has a 100-gon for its base, how many vertices, edges and faces does it have?



$$\begin{aligned} V &= 100 + 1 = 101 \\ e &= 100 + 100 = 200 \\ f &= 100 + 1 = 101 \end{aligned}$$

- 6 (b) If a pyramid has 14 edges, how many faces does it have?

$$7\text{-gon base} \Rightarrow f = 7 + 1 = 8$$

- 6 (c) Verify that your numbers in (a) and (b) satisfy Euler's formula.

$$V - e + f = 2$$

$$a.) \quad 101 - 200 + 101 = 2$$

$$b.) \quad 8 - 14 + 8 = 2$$

- 5 **Problem 3.** What fact about intersecting spheres enables three GPS satellites to determine your exact location on Earth?

3 spheres intersect in two points, one of which is on Earth.

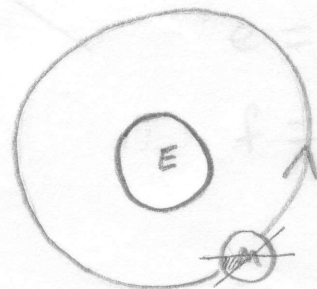
- 6 **Problem 4.** What fact about the interior angles of a regular octagon shows that a regular polyhedron cannot have octagonal faces?

$$\text{int. angle} = \frac{(n-2)180}{n} = \frac{6 \cdot 180}{8} = 135, \quad 3 \times 135 = 405 > 360$$

- 6 **Problem 5.** Consider the earth and moon as shown.

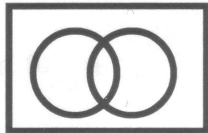
- (a) Is the moon new,  $\frac{1}{4}$ ,  $\frac{1}{2}$ ,  $\frac{3}{4}$ , or full?

- (b) Is it waxing or waning?



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**Problem 6.** For each of the following pairs, identify which type of Venn diagram describes their relationship.



overlapping



disjoint



subset

- (a) Concave shapes and polygons overlap
- (b) Convex shapes and parallelograms subset
- (c) Isosceles triangles and right triangles overlap
- (d) Rectangles and regular quadrilaterals subset
- (e) Platonic solids and pyramids overlap
- (f) Prisms and spheres disjoint
- (g) Parallelograms and rhombi subset
- (h) Kites and rhombi subset

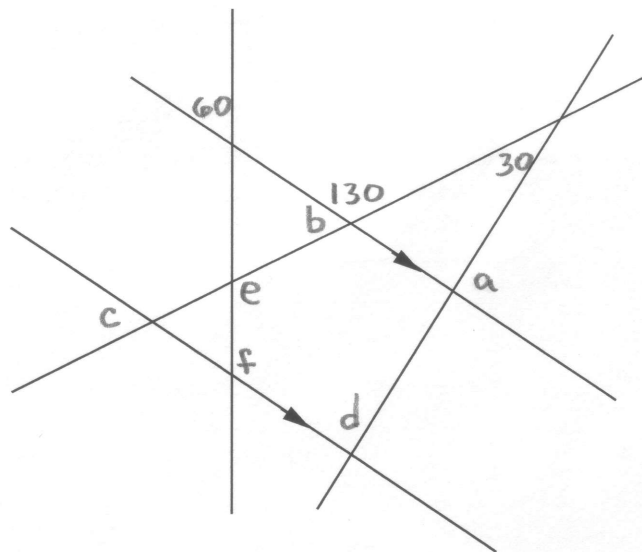
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**Problem 7.** Among parallelograms, rectangles, rhombi, and isosceles trapezoids, for which ones are the following statements always true:

- 4 (a) Diagonals are the same length. Rectangle, isosceles trap.
- 3 (b) Diagonals bisect angles. Rhombi
- 3 (c) Diagonals cross at right angles. Rhombi

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**Problem 8.** Find the missing angle measures indicated by letters in the diagram below. Two parallel lines are indicated by arrows.



- $a = 80$
- $b = 50$
- $c = 50$
- $d = 100$
- $e = 110$
- $f = 120$

Compass and straightedge constructions. Please do each one separately.

Problem 9. A \_\_\_\_\_ B

Given segment  $AB$ , construct an equilateral triangle with side  $AB$ .

Problem 10. Draw an angle that is approximately  $60^\circ$ . Precisely bisect this angle.

Problem 11. A \_\_\_\_\_ B

Given segment  $AB$ , draw a point  $C$  above it. Construct a line parallel to  $AB$  through  $C$  by copying an angle.

BONUS 12. A \_\_\_\_\_ B C \_\_\_\_\_ D

Construct a rectangle with given sides  $AB$  and  $CD$ .

30+10

