Calculus I (Math 231) Final Exam

Date: December 17, 2007 Professor Ilya Kofman

NAME: ____

Problem 1. Evaluate the following limits:

(a) $\lim\limits_{x\to 4} \dfrac{x^2-4x}{x^2-3x-4}$

(b) $\lim\limits_{x\to 0} \dfrac{8x}{\sin 2x}$

(c) $\lim\limits_{x\to -\infty} \dfrac{-5x^3+1}{17x^3+7x-11}$

Problem 2. Compute the first derivative for each of these functions:

(a) $f(x) = \dfrac{e^{3x}}{x^2+5}$

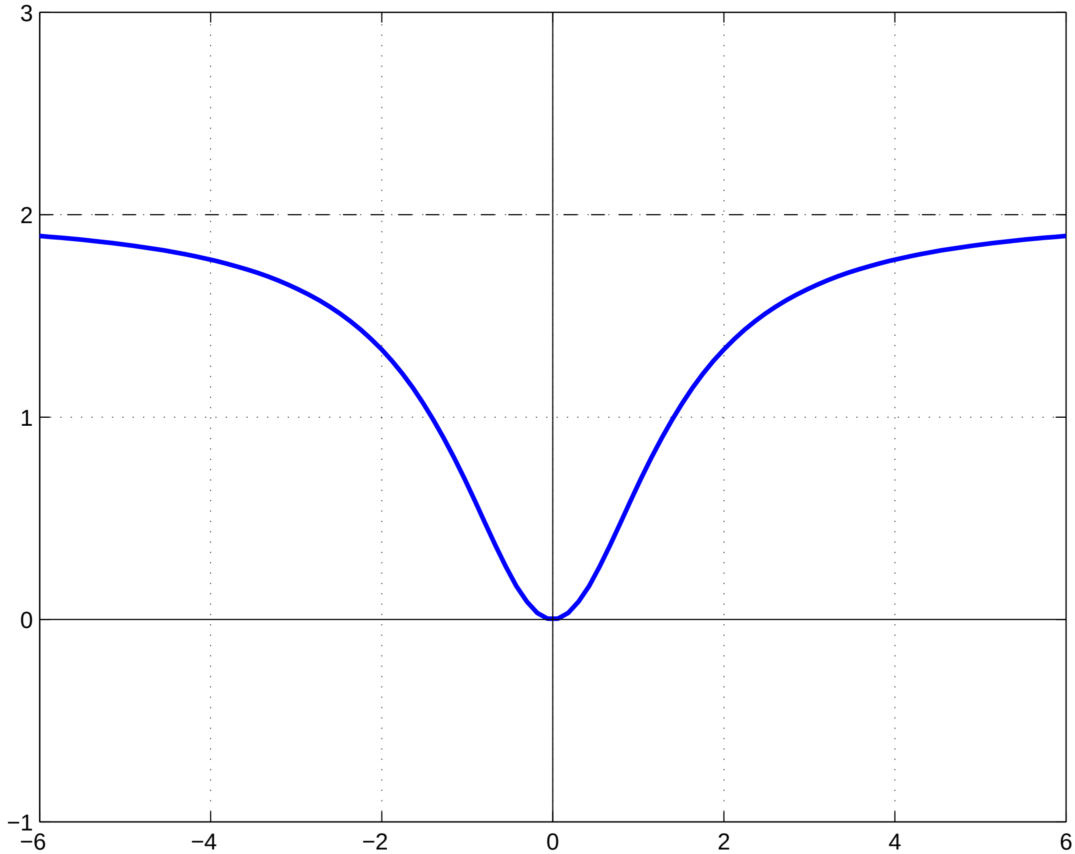
(b) $g(x) = \ln(6x)\sqrt{x^3+7x}$

Problem 3. Evaluate

(a) $\displaystyle\int \left(\frac{7}{x^4} + 3\sqrt{x} + e^{2x}\right) dx$

(b) $\displaystyle\int_1^3 \left(6x^2 + \frac{4}{x} + 5\right) dx$

Problem 4. Let $f(x)$ be the function defined by the following graph,



(a) $f'(x) < 0$ for which x? ________________________

$f'(x) > 0$ for which x? ________________________.

(b) $\lim_{x\to\infty} f(x) =$____________ and $\lim_{x\to-\infty} f(x) =$____________.

(c) Sketch a graph of $f'(x)$ on the figure.

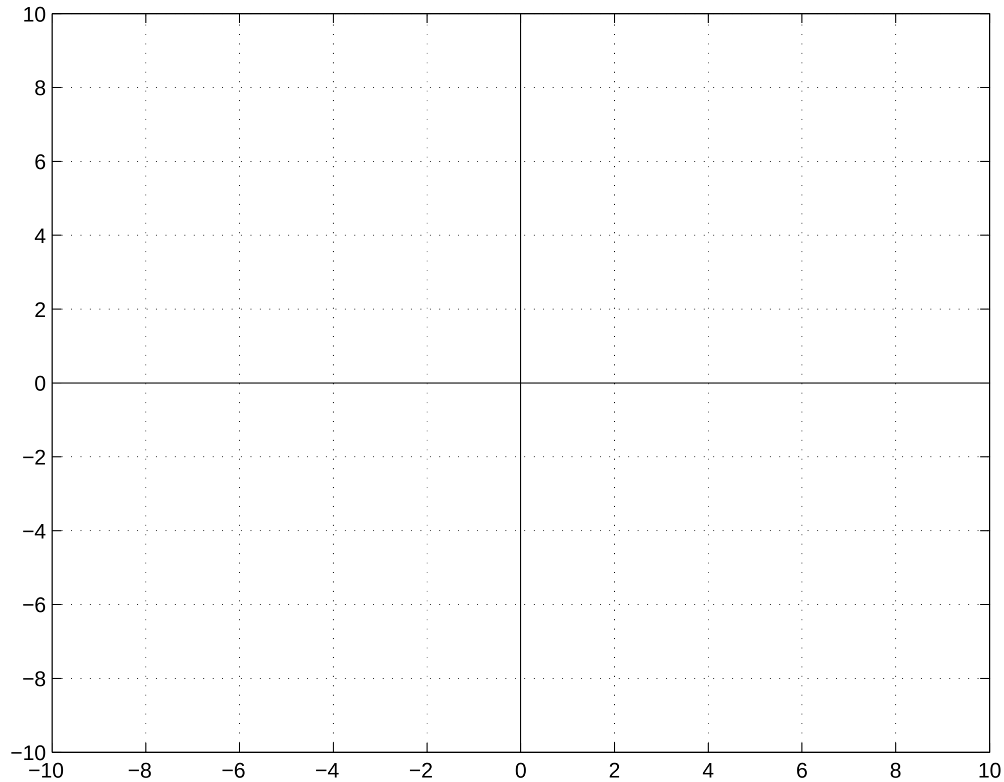
(d) Label the approximate locations of all points of inflection of $f(x)$.

(e) Sketch a graph of $f''(x)$ on the figure.

Make sure your sketches are clearly labeled above!

BONUS: $\lim_{x\to\infty} f'(x) =$____________ and $\lim_{x\to\infty} f''(x) =$____________.

Problem 5. Sketch the graph of a differentiable function $f(x)$ with all of the properties below.



- The domain of f is $(-\infty, -2) \cup (-2, \infty)$.
- $f(-6) = 1,\ f(-4) = -1$, and $f(3) = 0$.
- $\lim_{x \to -2} f(x) = \infty$.
- $\lim_{x \to -\infty} f(x) = 1$ and $\lim_{x \to \infty} f(x) = -\infty$.
- $f'(x) > 0$ for $-4 < x < -2$.
- $f'(x) < 0$ for $x < -4$ and for $x > -2$.
- $f''(x) > 0$ for $-6 < x < -2$ and for $-2 < x < 3$.
- $f''(x) < 0$ for $x < -6$ and for $x > 3$.

Label all horizontal and vertical asymptotes, local extrema, and inflection points.

Problem 6. Find the values of the constants m and b such that the following function is differentiable everywhere:

$$h(x) = \begin{cases} x^3 - 6x & \text{if } x \leq 2 \\ mx + b & \text{if } x > 2 \end{cases}$$

Problem 7. Answer questions below as True or False. (No explanation is needed.)

(a) ____________ The function $p(x) = \dfrac{|x|}{x}$ has a removable discontinuity at $x = 0$.

(b) ____________ The function $q(x) = 2x^5 - 10x$ has a zero in the interval $(1, 2)$.

(c) ____________ The function $r(x) = x^{1/3}$ has a vertical tangent line at the origin.

(d) ____________ If $s'(2) = 0$ then $x = 2$ is a local max or min of $s(x)$.

(e) ____________ A rational function can have at most two vertical asymptotes.

(f) ____________ $\displaystyle\int_0^5 f(x)\,dx = -\int_{-5}^0 f(x)\,dx$ for all integrable $f(x)$.

(g) ____________ $\displaystyle\frac{d}{dx}\left(\int_0^x t^{\sqrt{2}}\,dt\right) = x^{\sqrt{2}}.$

(h) ____________ $\displaystyle\int_0^{2\pi} |\sin x|\,dx = 2\int_0^{\pi} \sin x\,dx.$

(i) ____________ $\displaystyle\int_0^{\pi} \sin^2 x\,dx = \int_{\pi}^{2\pi} \sin^2 x\,dx.$

(j) ____________ $\displaystyle\int_{-4}^{4} (x^5 + 7x)^{13}\,dx = 0.$

CHOOSE ANY TWO PROBLEMS ON THIS PAGE

Problem 8. A paper cup in the shape of a circular cone has radius $r = 2\,\mathrm{cm}$ and height $h = 4\,\mathrm{cm}$. Water is poured into the cup at a rate of $2\,\mathrm{cm}^3/\mathrm{sec}$. Find the rate at which the water level is rising when the water is $3\,\mathrm{cm}$ deep. (Hint: $V = \frac{1}{3}\pi r^2 h$)

Problem 9. An open box with a total surface area of $300\ \mathrm{in}^2$ and with a square base is to be made from sheet metal. Find the dimensions of the box that will maximize its volume.

Problem 10. Consider the curve described by the relation $x^4 + y^4 = 32$. Find the equation of the tangent line to the curve at the point $(-2, 2)$.