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Business Calculus I (Math 221) Exam 3

Date: April 27, 2015 | Professor Ilya Kofman

Justify answers and show all work for full credit.

NAME: Key

Problem 1. Suppose x and y satisfy $4x^2 - 4 + y^6 = x^3 y - x + 7$.
Find $\frac{dy}{dx}$ at the point $(2, 1)$.

6 $$8x + 6y^5 \frac{dy}{dx} = \left(x^3 \frac{dy}{dx} + 3x^2 y\right) - 1$$

$$8(2) + 6(1)\frac{dy}{dx} = (2)^3 \frac{dy}{dx} + 3(2)^2(1) - 1$$

2 $$-2\frac{dy}{dx} = -5 \Rightarrow \frac{dy}{dx} = \frac{5}{2}$$

Problem 2. Find the derivatives $\frac{dy}{dx}$.

(a) $y = \ln(5x^3 + 4x + 1)$

4 $$\frac{dy}{dx} = \frac{15x^2 + 4}{5x^3 + 4x + 1}$$

(b) $e^{-3y} = \ln(x^2) + 4y^3$

6 $$-3e^{-3y}\frac{dy}{dx} = \frac{2}{x} + 12y^2 \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{2/x}{-3e^{-3y} - 12y^2} = -\frac{2}{x(3e^{-3y} + 12y^2)}$$

(c) $x e^y = e^{(x^2)} + 3y$

6 $$\left(x e^y \frac{dy}{dx} + e^y\right) = 2xe^{x^2} + 3\frac{dy}{dx}$$

$$\frac{dy}{dx}\left(xe^y - 3\right) = 2xe^{x^2} - e^y \Rightarrow \frac{dy}{dx} = \frac{2xe^{x^2} - e^y}{xe^y - 3}$$

110 24

Problem 3. Evaluate

8

(a) $\int 12x^7 + \frac{4}{x^2} + \frac{3}{x} - 5\, dx$ $= \underbrace{\frac{12}{8} x^8}_{\frac{3}{2} x^8} - \frac{4}{x} + 3 \ln|x| - 5x + C$

8

(b) $\int 10x^{3/4} + 2e^{6x} + \sqrt[3]{x} + \frac{7}{\sqrt{x}}\, dx$ $= \int 10x^{3/4} + 2e^{6x} + x^{1/3} + 7x^{-1/2}\, dx$

$= 10\left(\frac{4}{7}\right) x^{7/4} + \frac{2}{6} e^{6x} + \frac{3}{4} x^{4/3} + 7(2) x^{1/2} + C$

$= \frac{40}{7} x^{7/4} + \frac{1}{3} e^{6x} + \frac{3}{4} x^{4/3} + 14 x^{1/2} + C$

6

(c) $\int \frac{x^6}{2x^7 + 9}\, dx$ $\quad u = 2x^7 + 9$

$du = 14x^6\, dx$

$= \frac{1}{14} \int \frac{1}{u}\, du = \frac{1}{14} \ln|u| + C = \frac{1}{14} \ln|2x^7 + 9| + C$

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(d) $\int (4x - 3)^{10}\, dx$ $\quad u = 4x - 3$

$du = 4\, dx$

$= \frac{1}{4} \int u^{10}\, du = \frac{1}{4} \cdot \frac{1}{11} u^{11} + C = \frac{1}{44} (4x-3)^{11} + C$

28

Problem 4. As you pour batter to make a circular pancake, the area increases at a rate of 2 cm^2/sec. How fast is the pancake radius increasing when the radius is 5 cm? ($A = \pi r^2$)

$A = \pi r^2$

2 $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$2 = 2\pi(5)\frac{dr}{dt}$

6 $\frac{dr}{dt} = \frac{2}{10\pi} = \frac{1}{5\pi}$ cm/sec

Problem 5. To produce x tarpies, the marginal cost in dollars is $\overline{MC} = 6x + 60$, and the marginal revenue is $\overline{MR} = 300$. The fixed cost for making tarpies is \$6000.

(a) Find the marginal profit function $\overline{MP}(x)$, where x is the number of tarpies.

(b) Find the profit function $P(x)$ for tarpies.

(c) After how many tarpies, if ever, will making tarpies be profitable? Explain.

4 a) $MP = MR - MC = 300 - (6x+60) = 240 - 6x$

6 b) $P = \int MP = \int 240 - 6x \, dx = 240x - 3x^2 + C$

$P(0) = -6000$

$P(x) = -3x^2 + 240x - 6000$

8 c) $P'(x) = 240 - 6x \overset{\text{set}}{=} 0 \Rightarrow x = 40$

This is a <u>max</u> since $P''(x) = -6 < 0$

$P(40) = -3(40)^2 + 240(40) - 6000$

$= -4800 + 9600 - 6000 = -1200$

So <u>never</u> profitable.

26

Problem 6. Suppose \$5,000 is invested paying 2.5% interest per year (APR).

(a) Find the amount after 4 years if interest is compounded continuously.

(b) How long will it take to have \$7,000 if interest is compounded continuously?

6 a) $P(t) = Pe^{rt} = 5000e^{0.025t}$

$P(4) = 5000e^{(0.025)(4)} = 5000e^{0.1} = \$5525.85$

8 b) $7000 = 5000e^{0.025t}$

$\frac{7}{5} = e^{0.025t}$

$\ln\left(\frac{7}{5}\right) = 0.025t$

$t = \ln\left(\frac{7}{5}\right) / 0.025 = 13.46$ yrs.

Problem 7. A mouse farm starts with 3 mice, and two months later has 18 mice. Assume the population growth continues exponentially.

(a) Find the function that models the population after t months.

(b) Find the population after 9 months.

(c) When will the population reach one million mice?

6 a) $P(t) = P_0 e^{kt} = 3e^{kt}$

$18 = 3e^{k(2)} \Rightarrow 6 = e^{2k}$

$\ln 6 = 2k \Rightarrow k = \frac{1}{2}\ln 6 = 0.89588$

6 b) $P(9) = 3e^{(0.89588)(9)} = 9523$

6 c) $1{,}000{,}000 = 3e^{(0.89588)t}$

$\frac{1000000}{3} = e^{(0.89588)t}$

$\ln\left(\frac{1000000}{3}\right) = (0.89588)t \Rightarrow t = 14.2$ months

32