Business Calculus I (Math 221) Exam 3

Date: April 27, 2015 Professor Ilya Kofman

Justify answers and show all work for full credit.

NAME: _____

Problem 1. Suppose x and y satisfy $4x^2 - 4 + y^6 = x^3\, y - x + 7$.
Find $\dfrac{dy}{dx}$ at the point $(2, 1)$.

Problem 2. Find the derivatives $\dfrac{dy}{dx}$.

(a) $y = \ln(5x^3 + 4x + 1)$

(b) $e^{-3y} = \ln(x^2) + 4y^3$

(c) $x\, e^y = e^{(x^2)} + 3y$

Problem 3. Evaluate

(a) $\int 12x^7 + \frac{4}{x^2} + \frac{3}{x} - 5 \, dx$

(b) $\int 10x^{3/4} + 2e^{6x} + \sqrt[3]{x} + \frac{7}{\sqrt{x}} \, dx$

(c) $\int \frac{x^6}{2x^7 + 9} \, dx$

(d) $\int (4x - 3)^{10} \, dx$

Problem 4. As you pour batter to make a circular pancake, the area increases at a rate of 2 cm^2/sec. How fast is the pancake radius increasing when the radius is $5\,\text{cm}$? ($A = \pi r^2$)

Problem 5. To produce x tarpies, the marginal cost in dollars is $\overline{MC} = 6x + 60$, and the marginal revenue is $\overline{MR} = 300$. The fixed cost for making tarpies is \$6000.

(a) Find the marginal profit function $\overline{MP}(x)$, where x is the number of tarpies.

(b) Find the profit function $P(x)$ for tarpies.

(c) After how many tarpies, if ever, will making tarpies be profitable? Explain.

Problem 6. Suppose \$5,000 is invested paying 2.5% interest per year (APR).

(a) Find the amount after 4 years if interest is compounded continuously.

(b) How long will it take to have \$7,000 if interest is compounded continuously?

Problem 7. A mouse farm starts with 3 mice, and two months later has 18 mice. Assume the population growth continues exponentially.

(a) Find the function that models the population after t months.

(b) Find the population after 9 months.

(c) When will the population reach one million mice?