Business Calculus I (Math 221) Exam 2

Date: April 1, 2015 Professor Ilya Kofman

Justify answers and show all work for full credit. No calculators allowed.

NAME: Key

Problem 1. Let $f(x) = -\frac{3}{5}x^5 + \frac{3}{4}x^4 + 20x^3 - 5$.

(a) Find the critical points.

(b) Find intervals where $f(x)$ is increasing or decreasing.

(c) Identify all relative extrema and saddle points using the First Derivative Test.

6 a) $f'(x) = -3x^4 + 3x^3 + 60x^2$

$= -3x^2(x^2 - x - 20) = -3x^2(x-5)(x+4) \overset{\text{set}}{=} 0$

CP: $x = 0$, $x = -4$, $x = 5$

8 b)

		-4		0		5	
$-3x^2$	$-$		$-$		$-$		$-$
$x-5$	$-$		$-$		$-$		$+$
$x+4$	$-$		$+$		$+$		$+$
$f'(x)$	$-$		$+$		$+$		$-$
$f(x)$	dec		inc		inc		dec

$f(x)$ increasing $(-4, 0) \cup (0, 5)$

$f(x)$ decreasing $(-\infty, -4) \cup (5, \infty)$

6 c) $x = -4$ local min

$x = 0$ saddle point

$x = 5$ local max

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Problem 2. Let $f(x) = \frac{1}{4}x^4 - \frac{15}{2}x^2 + 3$.

(a) Find the critical points.

(b) Find intervals where $f(x)$ is concave up or down.

(c) Find the inflection points.

(d) Identify all relative extrema using the Second Derivative Test.

4 a) $f'(x) = x^3 - 15x = x(x^2-15) = x(x-\sqrt{15})(x+\sqrt{15}) \overset{set}{=} 0$

CP: $x=0,\ x=-\sqrt{15},\ x=\sqrt{15}$

8 b) $f''(x) = 3x^2 - 15 = 3(x^2-5) = 3(x-\sqrt{5})(x+\sqrt{5})$

		$-\sqrt{5}$		$\sqrt{5}$	
$x-\sqrt{5}$	$-$		$-$		$+$
$x+\sqrt{5}$	$-$		$+$		$+$
$f''(x)$	$+$		$-$		$+$
$f(x)$	CU		CD		CU

$f(x)$ Concave up $(-\infty, -\sqrt{5}) \cup (\sqrt{5}, \infty)$

$f(x)$ Concave down $(-\sqrt{5}, \sqrt{5})$

2 c) PI at $x=-\sqrt{5}$ and $x=\sqrt{5}$

6 d) $f''(0) = -15 < 0$ $x=0$ local max

$f''(-\sqrt{15}) = 30 > 0$ $x=-\sqrt{15}$ local min

$f''(\sqrt{15}) = 30 > 0$ $x=\sqrt{15}$ local min

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Problem 3. Find the absolute max and min: $f(x) = x^3 - 12x + 1, \ -1 \le x \le 3$.

2

$f'(x) = 3x^2 - 12 = 3(x^2-4) = 3(x-2)(x+2) \overset{\text{set}}{=} 0$

CP: $x = -2$ and $x = 2$

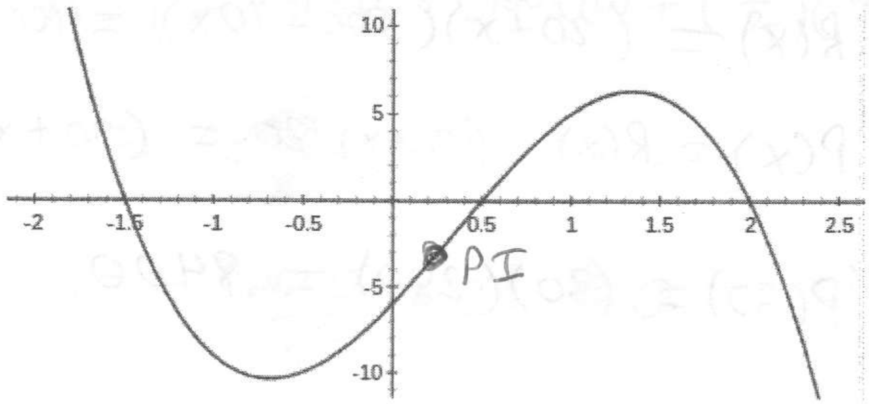
4

$f(-1) = 12 \Rightarrow$ Absolute max at $x = -1$

$f(2) = -15 \Rightarrow$ Absolute min at $x = 2$

$f(3) = -8$

Problem 4. The graph $y = f'(x)$ of the <u>derivative</u> of $f(x)$ is shown below.



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(a) Label all inflection points on the graph above with "PI".

2

(b) What are the critical points of $f(x)$?

$x = -1.5, \quad x = 0.5, \quad x = 2$

3

(c) On what intervals is $f(x)$ increasing?

$(-\infty, -1.5) \cup (0.5, 2)$

3

(d) On what intervals is $f(x)$ decreasing?

$(-1.5, 0.5) \cup (2, \infty)$

6

(e) Identify critical points of $f(x)$ as local max or min. Justify your answers.

$x = -1.5$	$x = 0.5$	$x = 2$
f' + | −	f' − | +	f' + | −
f inc dec.	f dec incr	f incr dec
local max	local min	local max

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Problem 5. An agency plans tours for groups of 20 or more. For 20 people, the price is \$500 per person. Each person's price is reduced by \$10 for each additional person in the group above 20. The agency's cost is \$120 per person.
Hint: Let x be the number of people in the group above 20.

(a) What is the revenue function $R(x)$?

(b) What is the profit function $P(x)$?

(c) What is the profit for a group of 30?

(d) What size group will give the agency the maximum profit?

(e) Justify using calculus that your price in part (c) gives the maximum profit.

3 a) $R(x) = (20+x)(500-10x) = 10000 + 300x - 10x^2$

3 b) $P(x) = R(x) - (20+x)120 = (20+x)(380-10x)$

2 c) $P(30) = (30)(280) = 8400$

6 d) $P'(x) = (20+x)(-10) + (380-10x)$

$= 180 - 20x \overset{\text{set}}{=} 0$

CP: $x = 9$, max profit at group size $\underline{29}$

4 e) Either $0 \le x \le 38$

$P(0) = (20)(380)$

$P(9) = (29)(290)$ $\underline{\text{Max}}$

$P(38) = (58)(0)$

$\underline{\text{OR}}$ $P''(x) = -20 < 0$

so $x = 9$ c.p. is a max

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Problem 6. A company needs 400 items per year. Production costs are \$50 for a production run, and \$10 per item. Inventory costs are \$4 per item per year. Hint: Let x be the number of items in each production run.

(a) What is the total cost function $C(x)$ for both production and storage?

(b) Find the number of items that should be produced in each run so that the total cost is minimized.

(c) Find the minimum total cost.

(d) Explain using calculus why your answer in (b) gives the minimum total cost?

6 a) $C(x) = \left(\frac{400}{x}\right)(50) + (400)(10) + \left(\frac{x}{2}\right)(4)$

$= \frac{20000}{x} + 4000 + 2x$

6 b) $C'(x) = -\frac{20000}{x^2} + 2 \overset{set}{=} 0 \Rightarrow x^2 = 10000$

$\underline{x = 100}$ C.P.

4 c) $C(100) = 200 + 4000 + 200 = \underline{\$4400}$

4 d) Either $1 \le x \le 400$ OR $C''(x) = 40000x^{-3}$

$C(1) = 20000 + 4000 + 2$

$C(100) = 4400 \Rightarrow$ Min $C''(100) > 0$

$C(400) = 50 + 4000 + 800$ So $x = 100$ C.P. is a $\underline{\text{min}}$

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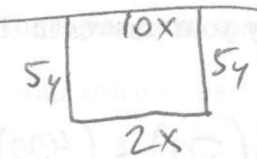
Problem 7. A rectangular field with one side along a road is to be fenced. The fence along the road costs \$10 per foot, the fence opposite the road costs \$2 per foot, and the fence perpendicular to the road costs \$5 per foot. The field must contain 120 square feet.

(a) Find the dimensions that minimize the total cost.

(b) Explain using calculus why your answer in part (a) gives the minimum cost?



a) $C(x,y) = 12x + 10y$

4 Constraint: Area $= xy = 120$

Let $y = \frac{120}{x}$

4 $C(x) = 12x + 10\left(\frac{120}{x}\right) = 12x + \frac{1200}{x}$

6 $C'(x) = 12 - \frac{1200}{x^2} \overset{\text{set}}{=} 0 \Rightarrow x^2 = 100$

$\underline{x = 10}$

Then $y = \frac{120}{10} = 12$

Dimensions: $\underline{10 \times 12}$

4 b) $C''(x) = 2400x^{-3}$

$C''(10) > 0$ so $x = 10$ is local <u>min</u>

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