Business Calculus I (Math 221) Exam 2

Date: October 31, 2016 — Professor Ilya Kofman

Justify answers and show all work for full credit. No graphing calculators allowed.

NAME: Key

Problem 1. Let $f(x) = \frac{2}{5}x^5 - \frac{1}{2}x^4 - 8x^3 + 1$.

(a) Find the critical points.

(b) Find intervals where $f(x)$ is increasing or decreasing.

(c) Identify all relative extrema and saddle points using the First Derivative Test.

4 a) $f'(x) = 2x^4 - 2x^3 - 24x^2$

$= 2x^2(x^2 - x - 12) = 2x^2(x+3)(x-4)$

C.P. $x=0$, $x=-3$, $x=4$

8 b)

		-3		0		4	
$2x^2$	+		+		+		+
$x+3$	−		+		+		+
$x-4$	−		−		−		+
$f'(x)$	+		−		−		+
$f(x)$	inc		dec		dec		inc

$f(x)$ increasing on $(-\infty, -3) \cup (4, \infty)$

$f(x)$ decreasing on $(-3, 0) \cup (0, 4)$

6 c) $x=-3$ local max, $x=0$ saddle pt, $x=4$ local min

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Problem 2. Let $f(x) = -\frac{1}{4}x^4 + 9x^2 + 2$.

(a) Find the critical points.

(b) Find intervals where $f(x)$ is concave up or down.

(c) Find the inflection points.

(d) Identify all relative extrema using the Second Derivative Test.

4 a) $f'(x) = -x^3 + 18x = -x(x^2 - 18) = -x(x-\sqrt{18})(x+\sqrt{18})$

C.P. $x = 0$, $x = -3\sqrt{2}$, $x = 3\sqrt{2}$

8 b) $f''(x) = -3x^2 + 18 = -3(x^2 - 6) = -3(x-\sqrt{6})(x+\sqrt{6})$

		$-\sqrt{6}$		$\sqrt{6}$	
-3	$-$		$-$		$-$
$x-\sqrt{6}$	$-$		$-$		$+$
$x+\sqrt{6}$	$-$		$+$		$+$
$f''(x)$	$-$		$+$		$-$
$f(x)$	CD		CU		CD

$f(x)$ concave up $(-\sqrt{6}, \sqrt{6})$

$f(x)$ concave down $(-\infty, -\sqrt{6}) \cup (\sqrt{6}, \infty)$

2 c) PI at $x = -\sqrt{6}$ and $x = \sqrt{6}$

6 d) $f''(0) = 18 > 0$ $\cup$ $x = 0$ local min

$f''(-3\sqrt{2}) = -36 < 0$ $\cap$ $x = -3\sqrt{2}$ local max

$f''(3\sqrt{2}) = -36 < 0$ $\cap$ $x = 3\sqrt{2}$ local max

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Problem 3. Find the absolute max and min: $f(x) = 2x^3 - 15x^2 + 24x, \quad -1 \le x \le 2.$

$f'(x) = 6x^2 - 30x + 24 = 6(x^2 - 5x + 4) = 6(x-4)(x-1)$

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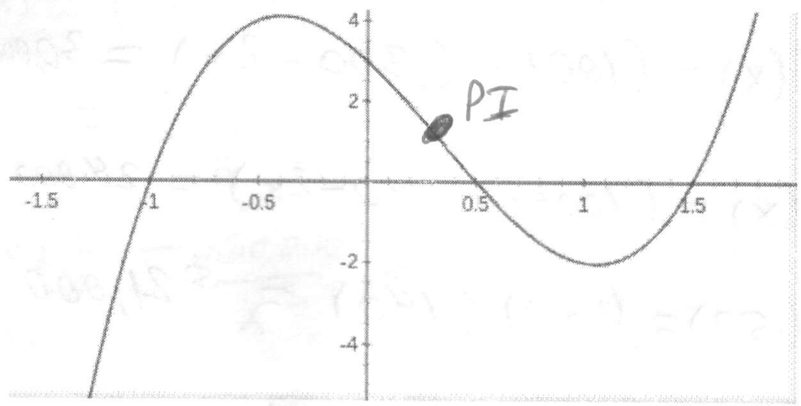
C.P. $x = 1, \ x = 4$

$f(-1) = -41 \Rightarrow$ Absolute min at $x=-1$

6 $f(1) = 11 \Rightarrow$ Absolute max at $x=1$

$f(2) = 4$

Problem 4. The graph $y = f'(x)$ of the <u>derivative</u> of $f(x)$ is shown below.



2 **(a)** Label all inflection points on the graph above with "PI".

2 **(b)** What are the critical points of $f(x)$?

$x = -1, \ x = 0.5, \ x = 1.5$

2 **(c)** On what intervals is $f(x)$ increasing?

$(-1, 0.5) \cup (1.5, \infty)$

2 **(d)** On what intervals is $f(x)$ decreasing?

$(-\infty, -1) \cup (0.5, 1.5)$

6 **(e)** Identify critical points of $f(x)$ as local max or min. Justify your answers.

$x=-1$	$x=0.5$	$x=1.5$
f': $-$ | $+$	f': $+$ | $-$	$-$ | $+$
f: dec, inc.	f: inc, dec.	dec, incr
local min	local max	local min

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Problem 5. A firm sells 100 monitors per month at \$300 each. From market research, the firm can sell one more monitor per month for each \$2 reduction in price. The firm's cost is \$60 per monitor.
Hint: Let x be the number of monitors sold per month above 100.

(a) What is the revenue function $R(x)$?

(b) What is the profit function $P(x)$?

(c) What is the profit if they sell 150 monitors per month?

(d) How many monitors should they sell to maximize profit?

(e) At what price will the profit be maximized? Justify using calculus that your price gives the maximum profit.

3 a) $R(x) = (100+x)(300-2x) = 30{,}000 + 100x - 2x^2$

3 b) $P(x) = (100+x)(240-2x) = 24{,}000 + 40x - 2x^2$

2 c) $P(50) = (150)(140) = \$21{,}000$

4 d) $P'(x) = 40 - 4x \overset{\text{set}}{=} 0$

C.P. $x=10$ They should sell 110 monitors

2 e) price at $x=10$ is $300-2x = \$280$

2 $p''(x) = -4 < 0$ so $x=10$ is max.

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Problem 6. A company needs 1,000 items per year. Production costs are \$160 for a production run, and \$12 per item. Inventory costs are \$8 per item per year. Hint: Let x be the number of items in each production run.

(a) What is the total cost function $C(x)$ for both production and storage?

(b) Find the number of items that should be produced in each run so that the total cost is minimized.

(c) Find the minimum total cost.

(d) Explain using calculus why your answer in (b) gives the minimum total cost?

6 a) $C(x) = \left(\frac{1000}{x}\right)(160) + (1000)(12) + \left(\frac{x}{2}\right)(8)$

$= \frac{160{,}000}{x} + 12{,}000 + 4x$

4 b) $C'(x) = -\frac{160{,}000}{x^2} + 4 \overset{\text{set}}{=} 0 \Rightarrow x^2 = 40{,}000$

$\underline{x = 200 \text{ C.P.}}$

4 c) $C(200) = 800 + 12{,}000 + 800 = \$13{,}600$

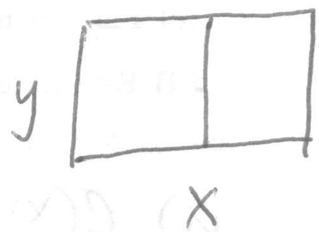
d) $C''(x) = \frac{320{,}000}{x^3}$

$C''(200) > 0$ so $x = 200$ is a min.

Problem 7. A rectangular field is to be fenced, with a divider down the middle to make two identical rectangular plots. The outside fence costs \$5 per foot, and the divider costs \$30 per foot. The field with both plots must contain 100 square feet in total.

(a) Find the dimensions that minimize the total cost.

(b) Explain using calculus why your answer in part (a) gives the minimum cost?



a) Area $= xy = 100$

$C(x,y) = 5(2x+2y) + 30y$

4 $\quad = 10x + 40y$

Let $x = \frac{100}{y}$

4 $\quad C(y) = 10\left(\frac{100}{y}\right) + 40y = \frac{1000}{y} + 40y$

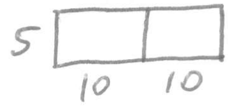
$C'(y) = -\frac{1000}{y^2} + 40 \overset{\text{set}}{=} 0 \Rightarrow 40y^2 = 1000$

$y^2 = 25$

$\underline{y = 5}$

Then $x = \frac{100}{5} = 20$

6 $\quad$ Dimensions: 5×20



b) $C''(y) = \frac{2000}{y^3}$

2 $\quad C''(5) > 0$ so $y = 5$ is <u>min</u>.

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