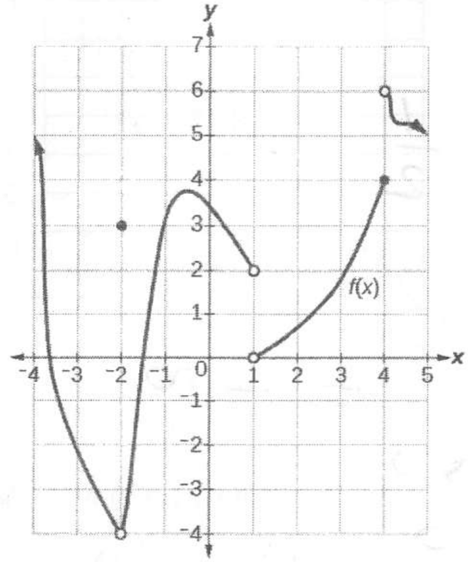
Business Calculus I (Math 221) Exam 1

March 4, 2015 — Professor Ilya Kofman

Justify answers and show all work for full credit. No calculators permitted on this exam.

NAME: Key



Problem 1 (20pts). *The graph of $y = f(x)$ is shown above. Evaluate each limit, or write DNE if the limit does not exist. No justifications are necessary for this problem.*

(a) $\lim_{x\to -2} f(x) =$ -4

(b) $\lim_{x\to 1^-} f(x) =$ 2

(c) $\lim_{x\to 1} f(x) =$ DNE

(d) $\lim_{x\to -3} f(x) =$ -2

(e) $\lim_{x\to 4^+} f(x) =$ 6

(f) $\lim_{x\to 4^-} f(x) =$ 4

(g) *For $f(x)$ to be continuous at $x = -2$, we must set $f(-2) =$* -4

(h) *Estimate the derivative* $f'(0) =$ -1

(i) *Estimate the derivative* $f'(3.5) =$ 2

(j) *Estimate for which x the derivative $f'(x) = 0$,* $x =$ $-\frac{1}{2}$

110

20

Problem 2 (12pts). *Evaluate these limits. For an infinite limit, write $+\infty$ or $-\infty$. If a limit does not exist (DNE), you must justify.* **Show all work!**

3 pts. each

(a) $\lim_{x\to 6} \frac{x^2-2x-24}{x^2-36}$ $= \lim_{x\to 6} \frac{\cancel{(x-6)}(x+4)}{\cancel{(x-6)}(x+6)} = \lim_{x\to 6} \frac{x+4}{x+6} = \frac{10}{12} = \frac{5}{6}$

(b) $\lim_{x\to 1^-} \frac{1}{x+1}$ $= \frac{1}{2}$

(c) $\lim_{x\to 1^-} \frac{1}{x-1}$ $= -\infty$

$(x-1) \to 0$

$x-1 < 0$

(d) $\lim_{x\to\infty} \frac{-8x^4+5x^2-2}{6x^4+3x^3-2x^2}$ $= \lim_{x\to\infty} \frac{-8+5/x^2-2/x^4}{6+3/x-2/x^2} = -\frac{8}{6} = -\frac{4}{3}$

Problem 3 (8pts). *Recall* $f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$.

3 (a) *If $f(x)=\sqrt{2x}$, write the limit for $f'(3)$. Do not evaluate this limit.*

$$f'(3) = \lim_{h\to 0} \frac{\sqrt{2(3+h)} - \sqrt{6}}{h}$$

5 (b) *Show that $g(x)=|x|$ is not differentiable at 0. Evaluate this limit.* **Show all work!**

$$g'(0) = \lim_{h\to 0} \frac{|0+h|-|0|}{h} = \lim_{h\to 0} \frac{|h|}{h}$$

$$\lim_{h\to 0^+} \frac{|h|}{h} = \lim_{h\to 0^+} \frac{h}{h} = 1 \quad \Rightarrow \quad g'(0) = \lim_{h\to 0} \frac{|h|}{h} \ \underline{\text{DNE}}$$

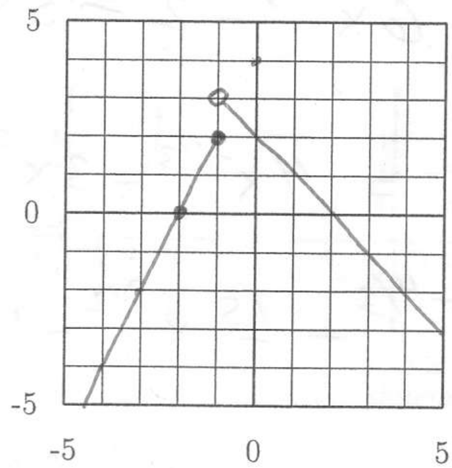
$$\lim_{h\to 0^-} \frac{|h|}{h} = \lim_{h\to 0^-} \frac{-h}{h} = -1$$

20

Problem 4 (5pts). **(a)** *On the grid below, graph the following piecewise defined function.*

$$f(x) = \begin{cases} 4 + 2x & x \leq -1 \\ 2 - x & x > -1 \end{cases}$$

(b) *Is the function $f(x)$ continuous at $x = -1$? (Do not justify.)* **YES** **NO** 1



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Problem 5 (6pts). *For what value of c (if any) is the function $g(x)$ continuous at $x = 2$? Justify your answer.*

$$g(x) = \begin{cases} x^3 - \dfrac{2x-1}{3} & x < 2 \\ c & x = 2 \\ x^2 + \dfrac{3x}{2} & x > 2 \end{cases}$$

$$\lim_{x\to 2^-} g(x) = \lim_{x\to 2^-} x^3 - \frac{2x-1}{3} = 8 - \frac{3}{3} = 7$$

$$\lim_{x\to 2^+} g(x) = \lim_{x\to 2^+} x^2 + \frac{3x}{2} = 4 + \frac{6}{2} = 7$$

$\Rightarrow \lim_{x\to 2} g(x) = 7$ 4

So let $g(2) = c = 7$ 2

Then $g(2) = \lim_{x\to 2} g(x)$

11

4 pts. each

Problem 6 (24pts). *Compute the derivative* $y' = \dfrac{dy}{dx}$. *Do not simplify. Show all work!*

(a) $y = \dfrac{x^3}{2} + 9x^{2/3} - 2x + 6 + 10x^{-1/2}$

$$y' = \frac{3}{2}x^2 + 6x^{-1/3} - 2 - 5x^{-3/2}$$

(b) $y = \dfrac{4}{\sqrt[3]{x}} - 3\sqrt{x^5} + \dfrac{10}{x} + \dfrac{5}{x^6}$ $= 4x^{-1/3} - 3x^{5/2} + 10x^{-1} + 5x^{-6}$

$$y' = -\frac{4}{3}x^{-4/3} - \frac{15}{2}x^{3/2} - 10x^{-2} - 30x^{-7}$$

(c) $y = \sqrt{5x^3 - 4x^2 - 3}$

$$y' = \frac{1}{2}(5x^3 - 4x^2 - 3)^{-1/2}(15x^2 - 8x)$$

(d) $y = \dfrac{8x^4 + 7x^3}{x^6 - 3}$

$$y' = \frac{(x^6-3)(32x^3+21x^2) - (8x^4+7x^3)(6x^5)}{(x^6-3)^2}$$

(e) $y = (3x^4 + 2x^3 + 7)(5x^9 - 8)$

$$y' = (3x^4 + 2x^3 + 7)(45x^8) + (12x^3 + 6x^2)(5x^9 - 8)$$

(f) $y = (6 + \sqrt[3]{x-4})^{-4/5}$

$$y' = -\frac{4}{5}\left(6 + (x-4)^{1/3}\right)^{-9/5}\left(\frac{1}{3}(x-4)^{-2/3}\right)$$

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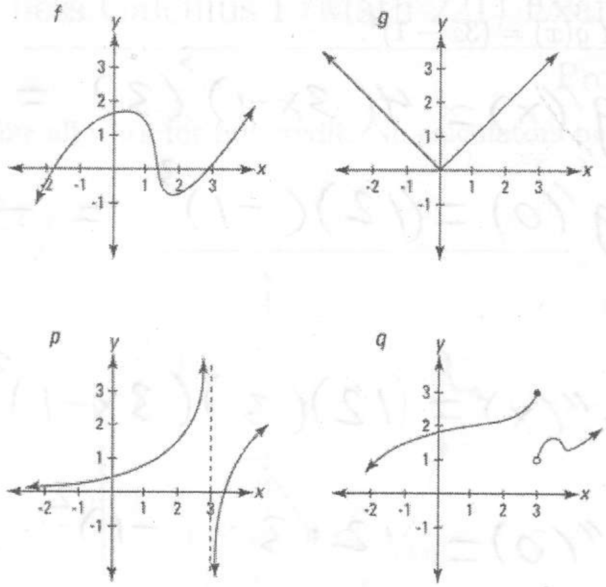

2 pts each

Problem 7 (10pts). *Circle every label for which the statement for that graph is true.*

(a) *The graph is continuous for all x shown.* f g p q

(b) *The graph is differentiable for all x shown.* f g p q

(c) *For some x shown, the derivative is zero.* f g p q

(d) *For all x where the derivative exists, it is positive.* f g p q

(e) *The derivative of the graph at $x = 0$ is positive.* f g p q

Problem 8 (5pts). *Let $F(x) = 2x^3 - x^2 + 1$. Find the equation of the tangent line to the graph of $F(x)$ at $x = 2$. Leave your answer in the form $y = mx + b$.*

$M = F'(2) = 6x^2 - 2x \Big|_{x=2} = 20$ 2

$F(2) = 16 - 4 + 1 = 13$ $(2, 13)$ 1

$y - 13 = 20(x - 2)$

$y = 20x - 40 + 13 \Rightarrow \underline{y = 20x - 27}$ 2

15

Problem 9 (8pts). *Let* $g(x) = (3x-1)^4$.

(a) *Find* $g'(0)$.

$g'(x) = 4(3x-1)^3(3) = 12(3x-1)^3$

$g'(0) = (12)(-1)^3 = -12$ 4

(b) *Find* $g''(0)$.

$g''(x) = (12)(3)(3x-1)^2(3)$

$g''(0) = 12 \cdot 3 \cdot (-1)^2 \cdot 3 = 108$ 4

Problem 10 (12pts). *For x units sold, the total revenue function is* $R(x) = 42x + 200$. *The total cost function is* $C(x) = 1000 + 30x + \frac{1}{5}x^2$.

(a) *Find the profit function* $P(x)$.

$P(x) = R(x) - C(x) = (42x + 200) - (1000 + 30x + \frac{1}{5}x^2)$

(b) *Find the marginal profit when 10 units are sold.* $P(x) = -\frac{1}{5}x^2 + 12x - 800$ 1

$P'(x) = -\frac{2}{5}x + 12$

$P'(10) = -\frac{2}{5}(10) + 12 = -4 + 12 = 8$ 4

(c) *If* $P(10) = -700$, *use your answer in part (b) to estimate the total profit if* 11 *units sold.*

$P(11) \approx P(10) + P'(10) = -700 + 8 = -692$

3

(d) *Should the company sell the 11th unit? Explain using your answers above.*

Yes, $P'(10) = 8 > 0$ Marginal profit is positive. 4

So profit will increase from sale of 11th unit.

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