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Business Calculus I (Math 221) Exam 1

September 28, 2016 — Professor Ilya Kofman

Justify answers and show all work for full credit. No calculators permitted on this exam.

NAME: Key

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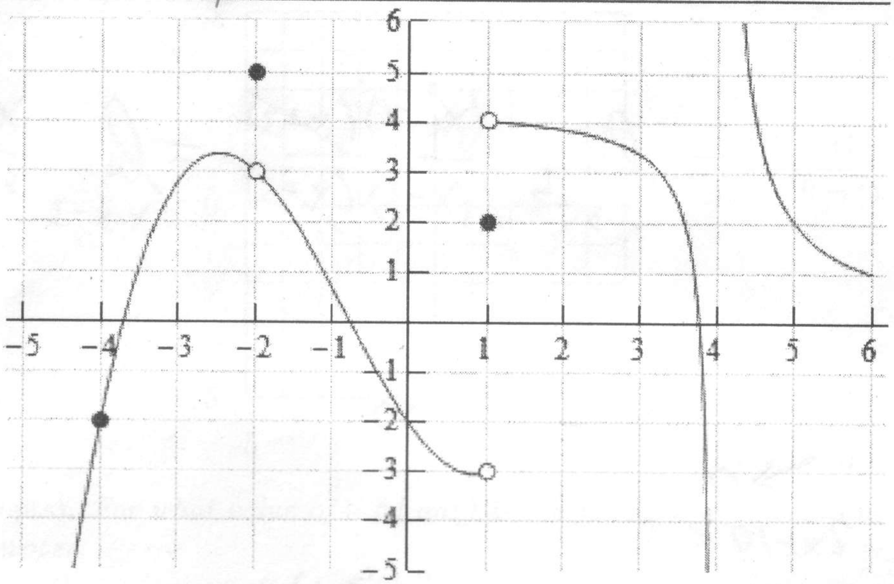


Problem 1 (20pts). *The graph of $y = f(x)$ is shown above. Evaluate each limit, or write DNE if the limit does not exist. No justifications are necessary for this problem.*

(a) $\lim_{x\to -2} f(x) =$ 3

(b) $\lim_{x\to -4} f(x) =$ −2

(c) $\lim_{x\to 1^+} f(x) =$ 4

(d) $\lim_{x\to 1^-} f(x) =$ −3

(e) $\lim_{x\to 1} f(x) =$ DNE

(f) $\lim_{x\to 4^-} f(x) =$ $-\infty$

(g) *For $f(x)$ to be continuous at $x = -2$, we must set $f(-2) =$* 3

(h) *Estimate the derivative* $f'(-1) =$ −1

(i) *Estimate the derivative* $f'(-3) =$ 1

(j) *Estimate for which x the derivative $f'(x) = 0$,* $x =$ −2.5

2pts each

110

20

Problem 2 (12pts). *Evaluate these limits. For an infinite limit, write* $+\infty$ *or* $-\infty$. *If a limit does not exist (DNE), you must justify.* **Show all work!**

3 pts. each

(a) $\lim_{x\to 4} \dfrac{x^2-13x+36}{x-4} = \lim_{x\to 4} \dfrac{(x-9)\cancel{(x-4)}}{\cancel{x-4}} = \lim_{x\to 4} x-9 = -5$

(b) $\lim_{x\to -3} \dfrac{x^2-4x-21}{x^2-9} = \lim_{x\to -3} \dfrac{(x-7)\cancel{(x+3)}}{(x-3)\cancel{(x+3)}} = \lim_{x\to -3} \dfrac{x-7}{x-3} = \dfrac{-10}{-6} = \dfrac{5}{3}$

(c) $\lim_{x\to 5^-} \dfrac{1}{2x-10} = -\infty$

$2x-10 \to 0$

$2x-10<0$

(d) $\lim_{x\to\infty} \dfrac{6x^5+8x^3-1}{-7x^5+3x^4-2x} = \lim_{x\to\infty} \dfrac{6+8/x^2-1/x^5}{-7+3/x-2/x^4} = -\dfrac{6}{7}$

Problem 3 (8pts). *Recall* $f'(a) = \lim_{h\to 0} \dfrac{f(a+h)-f(a)}{h}$.

3

(a) *If* $f(x)=4x^3$, *write the limit for* $f'(2)$. *Do not evaluate this limit.*

$$\lim_{h\to 0} \frac{4(2+h)^3-4(2)^3}{h} = \lim_{h\to 0} \frac{4(2+h)^3-32}{h}$$

5

(b) *Show that* $g(x)=|x|$ *is not differentiable at* 0. *Evaluate this limit.* **Show all work!**

$$g'(0) = \lim_{h\to 0} \frac{|0+h|-|0|}{h} = \lim_{h\to 0} \frac{|h|}{h}$$

$$\lim_{h\to 0^+} \frac{|h|}{h} = \lim_{h\to 0^+} \frac{h}{h} = 1$$

$\Rightarrow g'(0) = \lim_{h\to 0} \frac{|h|}{h}$ DNE

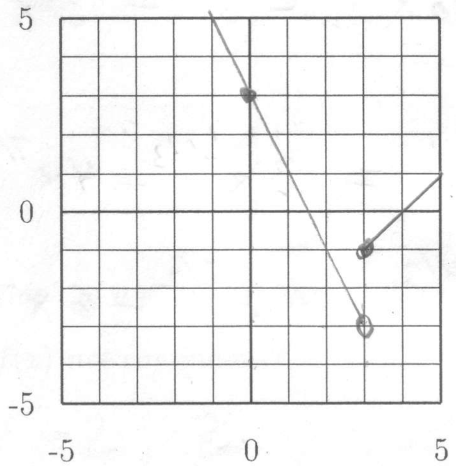
$$\lim_{h\to 0^-} \frac{|h|}{h} = \lim_{h\to 0^-} -\frac{h}{h} = -1$$

20

Problem 4 (5pts). **(a)** *On the grid below, graph the following piecewise defined function.*

$$f(x) = \begin{cases} 3 - 2x & x < 3 \\ x - 4 & x \geq 3 \end{cases}$$

(b) *Is the function $f(x)$ continuous at $x = 3$? (Do not justify.)* **YES** **NO** 1



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Problem 5 (6pts). *For what value of c (if any) is the function $g(x)$ continuous at $x = 4$? Justify your answer.*

$$g(x) = \begin{cases} \dfrac{x^2+2}{2x-2} & x < 4 \\ c & x = 4 \\ x^2 - 3x - 1 & x > 4 \end{cases}$$

$$\lim_{x \to 4^-} g(x) = \lim_{x \to 4^-} \frac{x^2+2}{2x-2} = \frac{18}{6} = 3$$ 4

$$\lim_{x \to 4^+} g(x) = \lim_{x \to 4^+} x^2 - 3x - 1 = 16 - 12 - 1 = 3$$

$$\Rightarrow \lim_{x \to 4} g(x) = 3$$

So let $g(4) = c = 3$ 2

Then $\lim_{x \to 4} g(x) = g(4)$

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Problem 6 (24pts). *Compute the derivative* $y' = \dfrac{dy}{dx}$. *Do not simplify. Show all work!*

4pts. each

(a) $y = \dfrac{x^4}{3} + 8x^{3/4} - 5x + 7 + 15x^{-1/5}$

$$y' = \frac{4}{3}x^3 + 6x^{-1/4} - 5 - 3x^{-6/5}$$

(b) $y = \dfrac{2}{\sqrt[3]{x}} - 4\sqrt{x^7} + \dfrac{6}{x} + \dfrac{3}{x^5}$ $= 2x^{-1/3} - 4x^{7/2} + 6x^{-1} + 3x^{-5}$

$$y' = -\frac{2}{3}x^{-4/3} - 14x^{5/2} - 6x^{-2} - 15x^{-6}$$

(c) $y = \sqrt[3]{3x^4 - 2x^3 - 4}$

$$y' = \frac{1}{3}(3x^4 - 2x^3 - 4)^{-2/3}(12x^3 - 6x^2)$$

(d) $y = \dfrac{6x^5 + 4x^3}{x^8 - 4}$

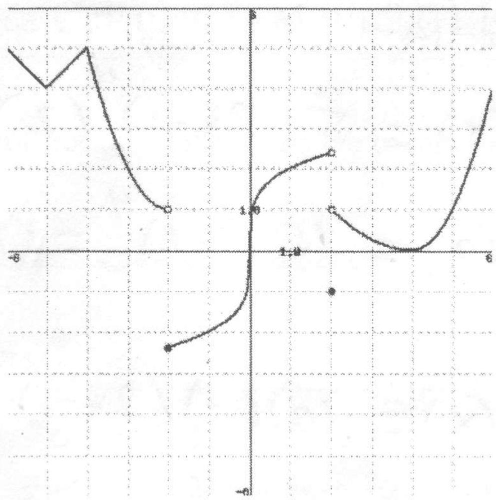
$$y' = \frac{(x^8-4)(30x^4+12x^2) - (6x^5+4x^3)(8x^7)}{(x^8-4)^2}$$

(e) $y = (4x^5 + 3x^4 + 20)(4x^9 - 10)$

$$y' = (4x^5+3x^4+20)(36x^8) + (20x^4+12x^3)(4x^9-10)$$

(f) $y = \sqrt{(3x-4)^5 - 10x}$

$$y' = \frac{1}{2}\left((3x-4)^5 - 10x\right)^{-1/2}\left(\underbrace{5(3x-4)^4(3)}_{15(3x-4)^4} - 10\right)$$



Problem 7 (~~10~~ 8 pts). *The graph of $y = f(x)$ is shown above for $-6 < x < 6$.*

(a) *For which x values is $f(x)$ not continuous?*

$x = -2, 2$ 2

(b) *For which x values is $f(x)$ not differentiable?*

$x = -5, -4, -2, 0, 2$ 4

(c) *For which x values is the derivative $f'(x) = 0$?*

$x = 4$ 2

Problem 8 ([illegible]pts). *Let $F(x) = 4x^3 - 2x^2 - 12$. Find the equation of the tangent line to the graph of $F(x)$ at $x = 1$. Leave your answer in the form $y = mx + b$.*

$m = F'(1) = (12x^2 - 4x)\Big|_{x=1} = 12 - 4 = 8$ 3

$F(1) = 4 - 2 - 12 = -10$ $(1, -10)$ 2

$y + 10 = 8(x - 1)$

$y = 8x - 18$ 2

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Problem 9 (8pts). *Let* $g(x) = (2x-1)^5$.

(a) *Find* $g'(0)$.

$g'(x) = 5(2x-1)^4(2) = 10(2x-1)^4$

$g'(0) = 10(-1)^4 = 10$ 4

(b) *Find* $g''(0)$.

$g''(x) = (10)(4)(2x-1)^3(2)$

$g''(0) = (10)(4)(-1)^3(2) = -80$ 4

Problem 10 (12pts). *For* x *units sold, the total revenue function is* $R(x) = 30x + 200$. *The total cost function is* $C(x) = 600 + 9x + \frac{1}{8}x^2$.

(a) *Find the profit function* $P(x)$.

$P(x) = R(x) - C(x) = (30x+200) - (600 + 9x + \frac{1}{8}x^2) = -\frac{1}{8}x^2 + 21x - 400$

(b) *Find the marginal profit when 100 units are sold.* 1

$P'(x) = -\frac{1}{4}x + 21$

$P'(100) = -\frac{1}{4}(100) + 21 = -25 + 21 = -4$ 4

(c) *If* $P(100) = 450$, *use your part (b) answer to estimate the total profit if 101 units sold.*

$P(101) \approx P(100) + P'(100) = 450 - 4 = 446$ 3

(d) *Should the company sell the 101st unit? Explain using your answers above.*

No, $P'(100) < 0$ Marginal profit is negative.
So Profit will decrease from sale of 101st unit. 4

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