Calculus I (Math 231) Exam 1

September 26, 2007 — Professor Ilya Kofman

Justify answers and show all work for full credit.

NAME: Solutions

Problem 1. *Compute these limits.* **For an infinite limit, write $+\infty$ or $-\infty$.** *Otherwise, if a limit does not exist (DNE), you must justify. Show all work!*

(a) $\lim\limits_{x\to-1} \dfrac{x^2-4x}{x^2-3x-4} = \dfrac{x(x-4)}{(x+1)(x-4)}$

$= \lim\limits_{x\to-1} \dfrac{\overset{\to -1}{x}}{\underset{\to 0}{x+1}}$ DNE (or $\pm\infty$)

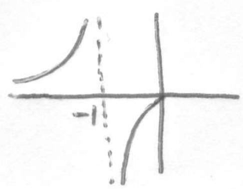
$\boxed{\#1a}$

Note:

$\lim\limits_{x\to-1^-} \dfrac{\overset{\to -1}{x}}{\underset{\to 0^-}{x+1}} = +\infty$

$\lim\limits_{x\to-1^+} \dfrac{\overset{\to -1}{x}}{\underset{\to 0^+}{x+1}} = -\infty$

Graph:



(b) $\lim\limits_{x\to0^-} \dfrac{\sqrt{1+x}-1}{x} \cdot \dfrac{\sqrt{1+x}+1}{\sqrt{1+x}+1} = \dfrac{(1+x)-1}{x(\sqrt{1+x}+1)}$

$= \lim\limits_{x\to0^-} \dfrac{\cancel{x}}{\cancel{x}(\sqrt{1+x}+1)} = \dfrac{1}{2}$

(c) $\lim\limits_{x\to-2} \dfrac{2-|x|}{2+x} = \dfrac{2+x}{2+x}$ ← Since $x<0$

$= \lim\limits_{x\to-2} \dfrac{2+x}{2+x} = 1$

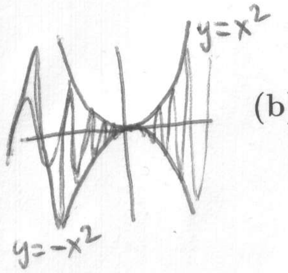
(d) $\lim\limits_{x\to0^-} \left(\dfrac{1}{x} - \dfrac{1}{|x|}\right) = \left(\dfrac{1}{x} + \dfrac{1}{x}\right)$ ← Since $x<0$

$= \lim\limits_{x\to0^-} \dfrac{2}{x} = -\infty$

Problem 2. *Compute and explain these limits.* **For an infinite limit, write $+\infty$ or $-\infty$.** *You must justify – show all work!*

(a) $\lim\limits_{x\to 0} \dfrac{\sin(7x)}{3x}$ $= \dfrac{7}{3} \cdot \dfrac{\sin(7x)}{7x}$

$$= \lim_{x\to 0} \frac{7}{3} \cdot \frac{\sin(7x)}{7x} = \frac{7}{3}\cdot 1 = \frac{7}{3}$$



(b) $\lim\limits_{x\to 0} x^2 \cos\left(\dfrac{2\pi}{x}\right)$

For all $x \neq 0$, $\left|\cos\left(\frac{2\pi}{x}\right)\right| \leq 1$

$= 0$

$$-1 \leq \cos\left(\frac{2\pi}{x}\right) \leq 1$$

by Squeeze Thm.

$$\underbrace{-x^2}_{\to 0} \leq x^2 \cos\left(\frac{2\pi}{x}\right) \leq \underbrace{x^2}_{\to 0} \text{ as } x \to 0$$

(c) $\lim\limits_{x\to 0} \left(\dfrac{1}{x\sqrt{1+x}} - \dfrac{1}{x}\right)$ $= \dfrac{1-\sqrt{1+x}}{x(\sqrt{1+x})}$

$$\lim_{x\to 0} \frac{1-\sqrt{1+x}}{x(\sqrt{1+x})} \cdot \frac{1+\sqrt{1+x}}{1+\sqrt{1+x}} = \frac{1-(1+x)}{x(\sqrt{1+x})(1+\sqrt{1+x})} = \frac{\cancel{-x}}{\cancel{x}(\sqrt{1+x})(1+\sqrt{1+x})}$$

$$= \frac{-1}{(1)(2)} = -\frac{1}{2}$$

(Bonus) Suppose $2x - 1 \leq f(x) \leq x^2$ for $0 < x < 3$. Find $\lim\limits_{x\to 1} f(x)$.

$$\lim_{x\to 1} 2x-1 = 1$$

$$\Rightarrow \lim_{x\to 1} f(x) = 1 \text{ by Squeeze Thm.}$$

$$\lim_{x\to 1} x^2 = 1$$

Problem 3. *Compute and explain these limits.* **For an infinite limit, write** $+\infty$ **or** $-\infty$. *You must justify – show all work!*

(a) $\lim_{x\to-\infty} \frac{2x^3+4x+1}{5x-3x^2}$ $= \lim_{x\to-\infty} \frac{2+4/x^2+1/x^3}{5/x^2-3/x}$ $\begin{matrix}\to 2\\ \to 0^+\end{matrix}$ $= +\infty$

(b) $\lim_{x\to\infty} \frac{(x+2)^2}{1+3x^2}$ $= \lim_{x\to\infty} \frac{x^2+4x+4}{3x^2+1} = \frac{1+4/x+4/x^2}{3+1/x^2} = \frac{1}{3}$

Problem 4. *Determine whether the function* $f(x)$ *is continuous at* $x = 2$.

$$f(x) = \begin{cases} \frac{6}{x} & 0 < x \le 2 \\ x^2 - \cos(\pi x) & x > 2 \end{cases}$$

1. $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} \frac{6}{x} = 3$

2. $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} x^2 - \cos(\pi x) = 4-1 = 3$

3. $f(2) = \frac{6}{2} = 3$ So yes, $f(x)$ cont. at $x=2$.

Problem 5. *For what value of the constant* c *is* $g(x)$ *everywhere continuous?*

$$g(x) = \begin{cases} cx^2+2x & x < 2 \\ x^3 - cx & x \ge 2 \end{cases}$$

$\lim_{x\to 2^-} g(x) = \lim_{x\to 2^-} cx^2+2x = 4c+4$

$\lim_{x\to 2^+} g(x) = \lim_{x\to 2^+} x^3 - cx = 8-2c$

$(g(2) = 20/3)$

$4c+4 = 8-2c$

$6c = 4$

$\underline{c = 2/3}$