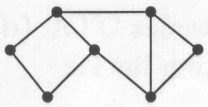
Mathematics for Liberal Arts (Math 102) Exam 1

Date: February 22, 2007 Professor Ilya Kofman

NAME: Key

Problem 1. *Answer the questions that are below each graph:*

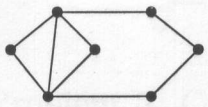


(a) *This graph has an Euler path, which is not a circuit.* T (F)

(b) *This graph has an Euler circuit.* T (F)

(c) *How many edges will a spanning tree for this graph have?* 6

(d) *How many edges must be <u>added</u> to best Eulerize this graph?* 2

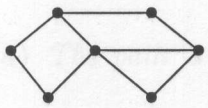


(e) *This graph has an Euler path, which is not a circuit.* T (F)

(f) *This graph has an Euler circuit.* (T) F

(g) *How many edges will a spanning tree for this graph have?* 6

(h) *How many edges must be <u>added</u> to best Eulerize this graph?* 0



(i) *This graph has an Euler path, which is not a circuit.* (T) F

(j) *This graph has an Euler circuit.* T (F)

(k) *How many edges will a spanning tree for this graph have?* 6

(l) *How many edges must be <u>added</u> to best Eulerize this graph?* 2

Problem 2. *Which <u>one</u> of the following techniques should be applied in each case:*

I. Apply Kruskal's algorithm to find minimal-cost spanning tree

II. Find an Euler circuit or best Eulerization

III. Apply the sorted-edges algorithm to solve TSP

(a) *NYC subway fan wants to quickly visit every station.* — *I* *II* (*III*) *None*

(b) *NYC subway fan wants to quickly see all graffiti on subway tunnel walls.* — *I* (*II*) *III* *None*

(c) *NYC wants to install new expensive fiber optic cable between all subway stations.* — (*I*) *II* *III* *None*

3 pts

Problem 3. *If a connected graph has* 14 *vertices of odd valence, at least how many edges must be added to Eulerize the graph?*

7

2 pts.

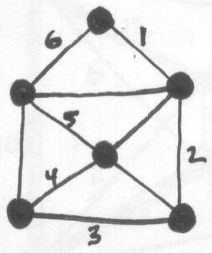
Problem 4. *Does the complete graph K_{13} have an Euler circuit? Why or why not?*

Yes, all valences = 12 (even)

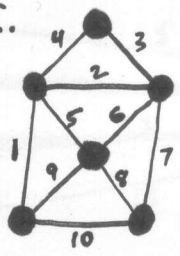
3 pts.

Problem 5. *Consider the paths given by the sequences of numbered edges on the graphs as shown. Circle the number of <u>every</u> graph whose path has the property:*

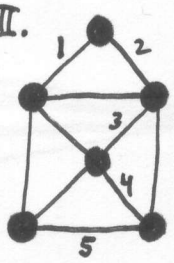
I.



II.



III.



(a) *The path is a circuit.* — (*I*) *II* *III* *None*

(b) *The path is a spanning tree.* — *I* *II* (*III*) *None*

(c) *The path covers every edge exactly once.* — *I* (*II*) *III* *None*

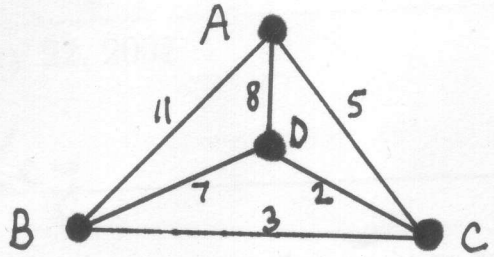
(d) *The path is an Euler circuit.* — *I* *II* *III* (*None*)

(e) *The path is a Hamiltonian circuit.* — (*I*) *II* *III* *None*

5 pts.

12

Problem 6. *For this graph, circle the correct answer below each question.*



(a) *Which routing is produced by the nearest-neighbor algorithm to solve TSP?*

1) *ABCDA* (2) *ABDCA* 3) *ACBDA* 4) *ACDAB* 5) *ABDAC*

(b) *Which routing is produced by the sorted-edges algorithm to solve TSP?*

(1) *ABCDA* 2) *ABDCA* 3) *ACBDA* 4) *ACDAB* 5) *ABDAC*

(c) *Which routing is produced by the brute-force algorithm to solve TSP?*

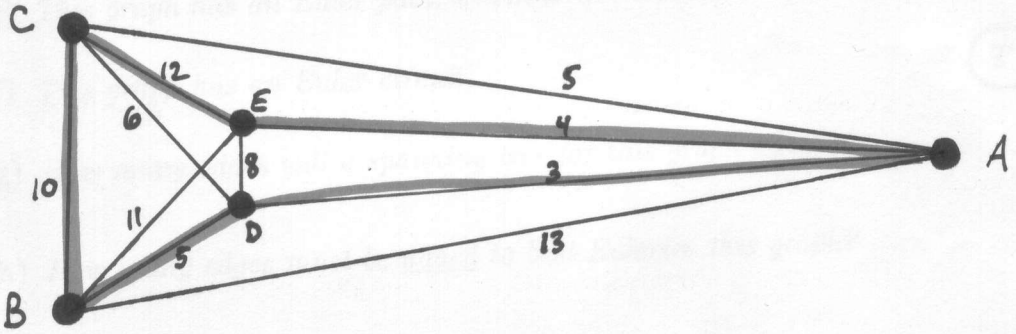
1) *ABCDA* 2) *ABDCA* (3) *ACBDA* 4) *ACDAB* 5) *ABDAC*

6 pts.

(d) *Using Kruskal's algorithm, what is the cost of the spanning tree?* 10

4 pts.

Problem 7. *Answer the questions for this graph. Show your work for full credit.*



(a) *What is the cost of the Hamiltonian circuit obtained by using the sorted-edges algorithm?*

3+4+5+10+12 = 34

Computed wrong circuit 2/5

5 pts.

(b) *What is the cost of the minimal-cost spanning tree using Kruskal's algorithm?*

3+4+5+5 = 17

Computed wrong tree 3/5

5 pts.

20