

**Math 233 Fall 2016 Sample Exam 1**

**Problem 1.** Consider three points  $P(2, -1, 0)$ ,  $Q(0, -2, 1)$  and  $R(3, 0, -1)$ .

- (a) Find a parametric equation of the line through  $Q$  and  $R$ .
- (b) Find the equation of the plane passing through  $P$ ,  $Q$ , and  $R$ .
- (c) Find the area of triangle  $\triangle PQR$ .

**Problem 2.** Find the equation of a plane containing the line  $\ell(t) = (2 + 3t, -t, 4 + t)$  and passing through the point  $P(0, 2, -1)$ .

**Problem 3.** The position of a particle is  $\mathbf{r}(t) = e^t \mathbf{i} + \sqrt{2}t \mathbf{j} + e^{-t} \mathbf{k}$ .

- (a) Show that the speed of the particle at time  $t$  is  $e^t + e^{-t}$ .
- (b) Find the unit tangent vector  $\mathbf{T}(t)$ .
- (c) Find the total distance travelled by the particle for  $1 \leq t \leq 3$ .

**Problem 4.** A string in the shape of a helix has a height of 15 cm and makes three full rotations over a circle of radius 4 cm.

- (a) Find a parametrization  $\mathbf{r}(t)$  for the string.
- (b) Compute the length of the string.

**Problem 5.** Show that if position  $\vec{\mathbf{r}}(t)$  satisfies  $\|\vec{\mathbf{r}}(t)\| = c$ , then velocity  $\vec{\mathbf{v}}(t)$  is orthogonal to  $\vec{\mathbf{r}}(t)$ .

**Problem 6.** Find the equation of the tangent plane to the ellipsoid  $2x^2 + 3y^2 + z^2 = 20$  at the point  $P(-2, 1, -3)$ .

**Problem 7.** Let  $S$  be the surface  $x^2 + 4y^2 - z^2 = 9$ .

- (a) Sketch the three traces of  $S$ , and then sketch  $S$ .
- (b) Find the equation of the tangent plane to  $S$  at the point  $P(-3, 2, 4)$ .

**Problem 8.**

- (a) Show that  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^3y}{2x^4 + y^4}$  does not exist.
- (b) Let  $h(x, y) = x \sin(x + 2y)$ . Verify Clairaut's Theorem:  $h_{xy} = h_{yx}$ .

**Problem 9.**

- (a) Show that  $\lim_{(x,y) \rightarrow (0,0)} \frac{|x|}{|x| + |y|}$  does not exist.
- (b) Let  $h(x, z) = e^{xz - x^2z^3}$ . Compute  $h_z(2, 1)$ .

**Problem 10.** For the function  $F(x, y, z) = x^2 - 2y^2 + z^2 + yz$

- (a) In what direction does the function increase fastest at the point  $P = (2, 1, -1)$ ?
- (b) What is the rate of fastest increase in part (a)?
- (c) What is the equation of the level surface of  $F$  through the point  $(2, 1, -1)$ ?
- (d) Find the equation of the tangent plane to the level surface of part (c) through the point  $(2, 1, -1)$ .