# Calculus III (Math 233) Exam 1

Date: September 28, 2016 Professor Ilya Kofman

Justify answers and show all work for full credit.

**NAME:** ________________________________________________

**Problem 1.** Consider three points $P(1,1,0)$, $Q(-2,1,0)$ and $R(0,-1,2)$.

**(a)** Find a parametric equation of the line through $P$ and $R$.

**(b)** Find the equation of the plane passing through $P$, $Q$, and $R$.

**(c)** Find the area of triangle $\triangle PQR$.

**(d)** Find the intersection of the plane and the line $\ell(t) = (2+3t,\ -t,\ 4+t)$.

**Problem 2.** The position of a particle is $\mathbf{r}(t) = (4\cos 3t, 5t+1, 4\sin 3t)$, for $0 \le t \le 2\pi$.

**(a)** Find the speed of the particle $v(t)$.

**(b)** Find the unit tangent vector $\mathbf{T}(t)$.

**(c)** Find the arclength for $0 \le t \le 2\pi$.

**(d)** Precisely describe the trajectory of the particle as a helix, including its height, base circle, number of revolutions, and direction.

**Problem 3.** Let $S$ be the surface $x^2 - 3y^2 + z^2 = 13$.

**(a)** Sketch the three traces of $S$, and then sketch $S$.

**(b)** Find the equation of the tangent plane to $S$ at the point $P(-4,2,3)$.

**Problem 4.** Assuming the earth is a round sphere, show that when you drive around in a car, your velocity vector is always tangent to the earth.

**Problem 5.**

**(a)** Show that $\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^4}$ does not exist.

**(b)** Compute all the first partial derivatives of $g(x, y, z) = \sqrt{5xy + 2z}$.

**(c)** Let $h(x, y, z) = \frac{x^2+y^2}{z^2+1}$. Compute $h_{xz}, h_{xy}, h_{yz}$.

**Problem 6.**

**(a)** Find the equation of the tangent plane for $f(x, y) = \log(2x^2 - 6y^2)$ at the point $P(2, 1)$.

**(b)** Find the equation of the tangent plane to the surface $xy - yz + zx = 6$ at the point $Q(2, 0, 3)$.

**Problem 7.** Suppose the plane $z = 3x - 4y + 7$ is tangent to the graph of $z = f(x, y)$ at the point $P(1, -2)$.

**(a)** Find the direction of maximum increase for the function $f(x, y)$ at $P$.

**(b)** Find $f(1, -2)$, and then approximate $f(1.02, -2.01)$.

**Problem 8.** The temperature at a point in the plane is $T(x, y) = 100 - 4x^3 - 3y^2$. A bug is at the point $(-1, 1)$.

**(a)** Compute $\nabla T(-1, 1)$.

**(b)** Find the rate of change of temperature in the direction of $\vec{v} = (3, 4)$.

**(c)** Find the direction in which the bug should move to increase its temperature the fastest.

**(d)** What is the maximum rate of increase of temperature?

**(e)** Find a direction in which the bug should move to NOT change its temperature.