Calculus III (Math 233) Exam 2

November 10, 2010 Professor Ilya Kofman

Justify answers and show all work for full credit.

NAME: ____

Problem 1. Evaluate $\iint_D x^2\, dA$ where D is the region bounded by the parabolas $y = 2x^2$ and $y = 1 + x^2$.

Problem 2. Find the volume enclosed by the paraboloid $z = 1 + 2x^2 + 2y^2$ and the plane $z = 7$.

Problem 3. Change the order of integration to integrate $\int_0^8 \int_{\sqrt[3]{y}}^2 \sin(x^4)\, dx\, dy$.

Problem 4. Let R be the region bounded by the lines $y + x = 0$ and $y + x = 5$, $y - x = 0$ and $y - x = 3$. Use the change of variables $x = \dfrac{u - v}{2}$ and $y = \dfrac{u + v}{2}$ (i.e., $u = x + y$ and $v = y - x$) to evaluate $\iint_R 2(x + y)\, dA$.

Problem 5. Completely set up, **but do not evaluate**, the following integrals:

(a) The volume of the tetrahedron bounded by the plane $3x + 2y + z = 6$ in the first octant.

(b) The volume of ice-cream bounded by the cone $z = \sqrt{x^2 + y^2}$ and the sphere $x^2 + y^2 + z^2 = 16$.

Problem 6. Evaluate $\iiint_E x^2 + y^2\, dV$ where E is the solid bounded by the paraboloids $z = 2 - x^2 - y^2$ and $z = x^2 + y^2 - 2$.

Problem 7. Use Lagrange multipliers to find the maximum and minimum values of the function $f(x, y) = x^2 + y^2 - 6x + 5$ on the ellipse $4x^2 + y^2 = 16$.

Problem 8. Find all the critical points of $f(x, y) = x^2 - y^2 + 4x + 6y - 16$, and classify them using the Second Derivative Test.

CHANGE OF VARIABLES FORMULAS:

$$\iint f(r, \theta)\, r\, dr\, d\theta, \text{ where } x = r\cos\theta,\ y = r\sin\theta$$

$$\iiint f(\rho, \phi, \theta)\ \rho^2 \sin\phi\ d\rho\ d\phi\ d\theta, \text{ where } x = \rho\sin\phi\cos\theta,\ y = \rho\sin\phi\sin\theta, z = \rho\cos\phi$$

$$\iint f(u, v) \left|\frac{\partial(x, y)}{\partial(u, v)}\right| du\, dv, \text{ where } \frac{\partial(x, y)}{\partial(u, v)} = \begin{vmatrix} \partial x/\partial u & \partial x/\partial v \\ \partial y/\partial u & \partial y/\partial v \end{vmatrix}$$